

Implicit Differentiation

ex. 1

$$x^2 + y^2 = 8 \quad \text{Find } \frac{dy}{dx} ?$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$$

eg. 2 $4x^2 - y^2 = 16$ Find $\frac{dy}{dx}$?

$$8x - 2y \frac{dy}{dx} = 0$$

$$-2y \frac{dy}{dx} = -8x$$

$$\frac{dy}{dx} = \frac{8x}{2y} = \frac{4x}{y}$$

eg. 3

$2x \cdot y^2 = 6$ Find $\frac{dy}{dx}$?

PRODUCT rule $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

$$\begin{array}{l} u = 2x \quad v = y^2 \\ \frac{du}{dx} = 2 \quad \frac{dv}{dx} = 2y \frac{dy}{dx} \end{array}$$

$$(2x)(2y \frac{dy}{dx}) + (2)(y^2) = 0$$

$$4xy \frac{dy}{dx} = -2y^2$$

$$\frac{dy}{dx} = \frac{-2y^2}{4xy} = \frac{-y}{2x}$$

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Q1 (v) Find $\frac{dy}{dx}$ if : $5x - x^2y + 2 = 0$ Derivative of $x^2y = ?$

$$u = x^2 \quad v = y \quad \Rightarrow \text{Derivative} = x^2 \frac{dy}{dx} + 2xy$$

$$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = \frac{dy}{dx}$$

$$\Rightarrow 5 - x^2 \frac{dy}{dx} - 2xy = 0$$

$$-x^2 \frac{dy}{dx} = 2xy - 5$$

$$x^2 \frac{dy}{dx} = 5 - 2xy$$

$$\frac{dy}{dx} = \frac{5 - 2xy}{x^2}$$