"Completing the Square"

eg.. $x^2 + 4x + 4$

factorise this $(x + 2)(x \cdot 2)$ two identical factors

$= (x+2)^2$

this is a complete square

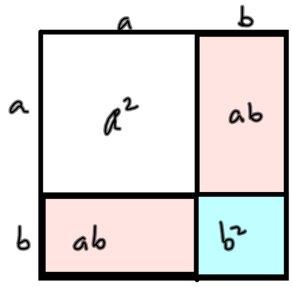
Consider: $(a+b)^2 = (a+b)(a+b)$

$= a^2 + ab + ab + b^2$ this is a complete square

$= a^2 + 2ab + b^2$

We could Represent this with a diagram

the 4 sections in the diagram



$\Rightarrow (a+b)^2 = a^2 + 2ab + b^2$

Now consider this quadratic

$$x^2 + 6x + 8$$

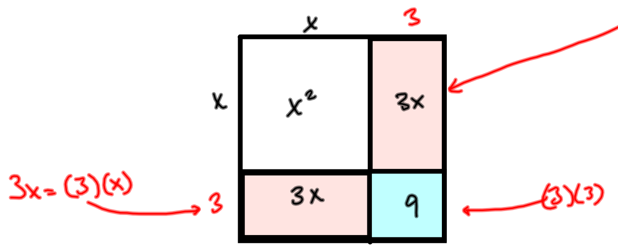
When we factorise $= (x+2)(x+4)$ factores are different

this is not a Complete Square

e.g.1 Complete the square: $x^2 + 6x + 8$

What complete square starts with "$x^2 + 6x$" ?

$6x = 3x + 3x$



$x^2 + 6x + 9 = (x+3)^2$ is a complete square related to $x^2 + 6x + 8$

so how can we write $x^2 + 6x + 8$ in terms of $(x+3)^2$

$$= \underline{x^2 + 6x + 9} + 8 - 9$$

$$= (x+3)^2 - 1$$

1. Find the value of c that completes the square in each of the following:

(i) $a^2 + 28a + c$

	a	14
a	a^2	$14a$
14	$14a$	196

$c = 196$

(ii) $x^2 - 6x + c$

	x	-3
x	x^2	$-3x$
-3	$-3x$	9

$c = 9$

(iii) $y^2 - 5y + c$

	y	$-5/2$
y	y^2	$-\frac{5}{2}y$
$-\frac{5}{2}$	$-\frac{5}{2}y$	$\frac{25}{4}$

$c = \frac{25}{4}$

2. Complete the square in each of the following:

(i) $x^2 - 8x - 3 = 0$

	x	-4
x	x^2	$-4x$
-4	$-4x$	16

$x^2 - 8x - 3 = 0$

$\underline{x^2 - 8x + 16} - 3 - 16 = 0$

$(x-4)^2 - 19 = 0$

(ii) $x^2 - 2x - 5 = 0$

	x	-1
x	x^2	$-x$
-1	$-x$	1

$x^2 - 2x - 5 = 0$

$\underline{x^2 - 2x + 1} - 5 - 1 = 0$

$(x-1)^2 - 6 = 0$

(iii) $x^2 - 2x + 1$

	x	-1
x	x^2	$-x$
-1	$-x$	1

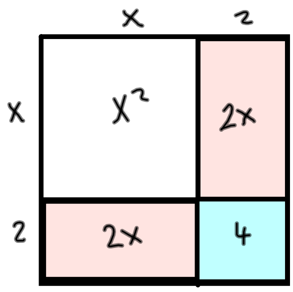
$x^2 + 2x + 1 = 0$

this is complete!

$(x-1)^2 = 0$

3. Write each of the following in the form $(x-p)^2+q=0$.

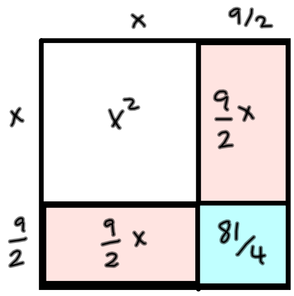
(i) $x^2+4x-6=0$ (ii) $x^2+9x+4=0$ (iii) $x^2-7x-3=0$



$x^2+4x-6=0$

$(x+2)^2-6-4=0$

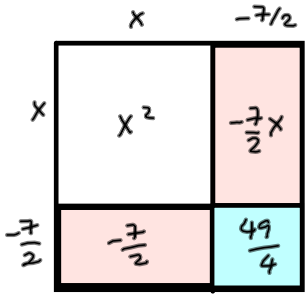
$(x+2)^2-10=0$



$x^2+9x+4=0$

$\left(x+\frac{9}{2}\right)^2+4-\frac{81}{4}=0$

$\left(x+\frac{9}{4}\right)^2-\frac{65}{4}=0$



$x^2-7x-3=0$

$\left(x-\frac{7}{2}\right)^2-3-\frac{49}{4}=0$

$\left(x-\frac{7}{2}\right)^2-\frac{61}{4}=0$

Why bother?

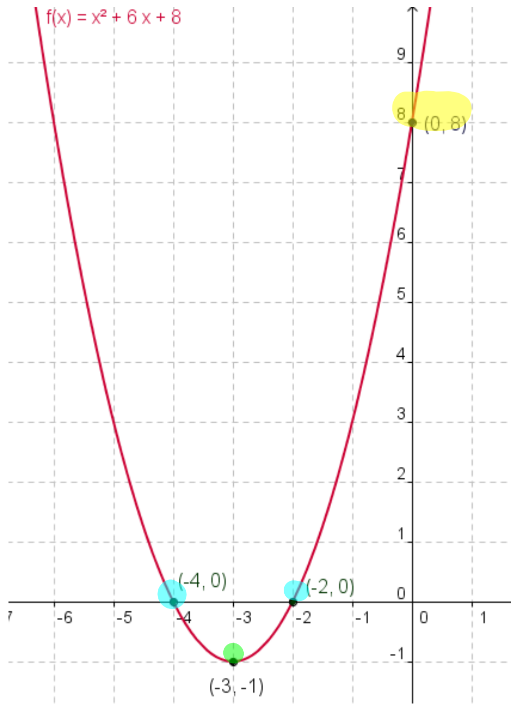
What is the purpose of writing

for example : $f(x)=x^2+6x+8$

in the complete square form : $f(x)=(x+3)^2-1$?

Its because it gives us the maximum or minimum point of the curve ie . $(-3,-1)$ is the minimum of the curve

note sign change.



If we look at the graph of $f(x) = x^2 + 6x + 8$

And consider 3 ways we might present this function:

① in form $ax^2 + bx + c$

$f(x) = x^2 + 6x + 8$ y-intercept $c=8$

② using factors

$f(x) = (x+2)(x+4)$ Roots at $x = -2, -4$

③ in complete square form

$f(x) = (x+3)^2 - 1$ minimum $(-3, -1)$

4. The graph of $y = a(x - p)^2 + q$ has a minimum point (p, q).
By completing the square, find the minimum point of each of the following quadratic equations:

(i) $2x^2 + 4x - 5 = 0$ (ii) $3x^2 - 6x - 1 = 0$ (iii) $4x^2 + x + 3 = 0$

(i)

$2\left[x^2 - 2x - \frac{5}{2}\right] = 0$

	x	-1
x	x^2	$-1x$
-1	$-1x$	1

$2\left[(x-1)^2 - \frac{5}{2} - 1\right] = 0$

$2\left[(x-1)^2 - \frac{7}{2}\right] = 0$

$2(x-1)^2 - 7 = 0$

min. $(1, -7)$

(ii)

$3\left[x^2 - 2x - \frac{1}{3}\right] = 0$

	x	-1
x	x^2	$-1x$
-1	$-1x$	1

$3\left[(x-1)^2 - \frac{1}{3} - 1\right] = 0$

$3\left[(x-1)^2 - \frac{4}{3}\right] = 0$

$3(x-1)^2 - 4 = 0$

min. $(1, -4)$

(iii)

$4\left[x^2 + \frac{1}{4}x + \frac{3}{4}\right] = 0$

	x	$\frac{1}{8}$
x	x^2	$\frac{1}{8}x$
$\frac{1}{8}$	$\frac{1}{8}x$	$\frac{1}{64}$

$4\left[(x+\frac{1}{8})^2 + \frac{3}{4} - \frac{1}{64}\right] = 0$

$4\left[(x+\frac{1}{8})^2 - \frac{47}{64}\right] = 0$

$4(x+\frac{1}{8})^2 - \frac{47}{16} = 0$

min. $(-\frac{1}{8}, -\frac{47}{16})$