Exercise 3.2

1. Write each of the following numbers in terms of i:

(i) $\sqrt{-4}$

$= \sqrt{(-1)(4)}$

$= \sqrt{-1}\sqrt{4}$

$= 2i$

(ii) $\sqrt{-36}$

$= \sqrt{(-1)(36)}$

$= \sqrt{-1}\sqrt{36}$

$= 6i$

(iii) $\sqrt{-27}$

$= \sqrt{(-1)(27)}$

$= \sqrt{-1}\sqrt{27}$

$= i\sqrt{27}$

(iv) $\sqrt{-20}$

$= \sqrt{(-1)(20)}$

$= \sqrt{-1}\sqrt{20}$

$= 2i\sqrt{5}$

3. Express each of the following in the form $a + bi$:

(i) $(3 + 2i) + (5 - i)$

$$\begin{array}{r} 3+2i \\ +2-i \\ \hline 5+i \end{array}$$

(ii) $(7 - 2i) + (3 - 4i)$

$$\begin{array}{r} 7-2i \\ +3-4i \\ \hline 10-6i \end{array}$$

(iii) $(-3 + 4i) + (6 - 4i)$

$$\begin{array}{r} -3+4i \\ +6-4i \\ \hline 3 \end{array}$$

(iv) $(-3 - i) + (-2 + 6i)$

$$\begin{array}{r} -3-i \\ -2+6i \\ \hline -5+5i \end{array}$$

(v) $(5 - 3i) + (-5 + 6i)$

$$\begin{array}{r} 5-3i \\ -5+6i \\ \hline 3i \end{array}$$

(vi) $(1 + i) + (2 - 3i)$

$$\begin{array}{r} 1+i \\ +2-3i \\ \hline 3-2i \end{array}$$

4. Simplify each of the following:

(i) $(2 + 6i) - (1 + 4i)$

$$\begin{array}{r} 2+6i \\ -1-4i \\ \hline 1+2i \end{array}$$

(ii) $(3 - 5i) - (2 + 4i)$

$$\begin{array}{r} 3-5i \\ -2-4i \\ \hline 1-9i \end{array}$$

(iii) $(4 - 7i) - (-1 + 3i)$

$$\begin{array}{r} 4-7i \\ +1-3i \\ \hline 5-10i \end{array}$$

(iv) $3 - (1 + 4i)$

$$\begin{array}{r} 3+0i \\ -1-4i \\ \hline 2-4i \end{array}$$

(v) $(3 - 6i) - 4i$

$$\begin{array}{r} 3-6i \\ -4i \\ \hline 3-10i \end{array}$$

(vi) $(-3 - 2i) - (4 - 7i)$

$$\begin{array}{r} -3-2i \\ -4+7i \\ \hline -7+5i \end{array}$$

5. Multiply each of the following complex numbers and give your answer in the form $a + bi, a, b \in R$:

(i) $(3 + 2i)(2 + 3i)$

$= 6 + 9i + 4i \mp 6\cancel{i^2}$

$= 0 + 13i$

(ii) $(4 + i)(3 - 5i)$

$= 12 - 20i + 3i \pm 5\cancel{i^2}$

$= 17 - 17i$

(iii) $(5 - 2i)(3 - 5i)$

$= 15 - 25i - 15i \mp 10\cancel{i^2}$

$= 5 - 40i$

(iv) $(3 + 4i)(3 - 4i)$

Diff. 2 Squares

$= 3^2 \pm 16\cancel{i^2}$

$= 9 + 16 = 25$

(v) $(5 - i)(5 + i)$

Diff. 2 Squares

$= 5^2 \pm 1\cancel{i^2}$

$= 25 + 1 = 26$

(vi) $(3 - 2i)^2$

$= 9 - 12i \mp 4\cancel{i^2}$

$= 5 - 12i$