Complex numbers

Section 3.1 Irrational numbers

Leaving Cert. Question 2011

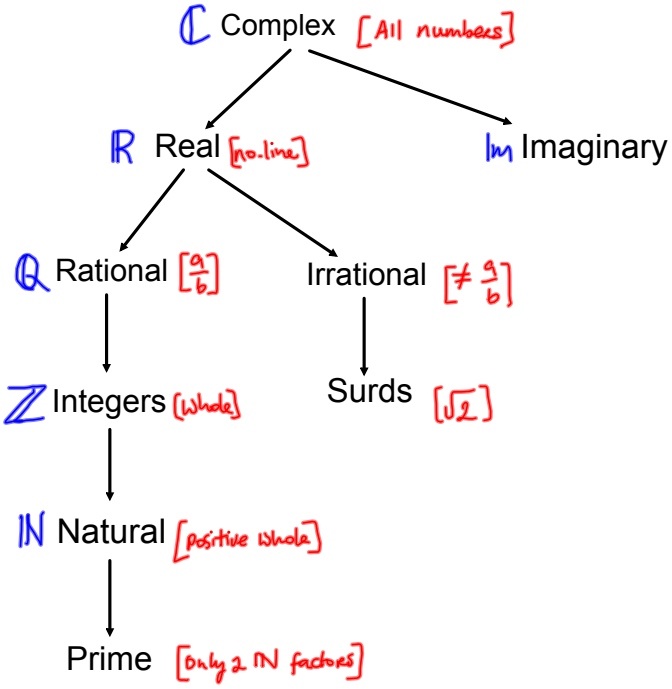
What is an irrational number - give an example?

It is a number that can't be written

as a fraction with 2 integers.

e.g. $\sqrt{2}$ or π

Number Systems



Imaginary Numbers

What is the imaginary unit?

$$i = \sqrt{-1}$$
$$\Rightarrow i^2 = -1$$

Section 3.2 Complex numbers

A Complex Number is a combination of a Real Number and an Imaginary Number

Exercise 3.2

1. Write each of the following numbers in terms of i:

(i) $\sqrt{-4}$

$= \sqrt{(-1)(4)}$

$= \sqrt{-1}\sqrt{4}$

$= 2i$

(ii) $\sqrt{-36}$

$= \sqrt{(-1)(36)}$

$= \sqrt{-1}\sqrt{36}$

$= 6i$

(iii) $\sqrt{-27}$

$= \sqrt{(-1)(27)}$

$= \sqrt{-1}\sqrt{27}$

$= i\sqrt{27}$

(iv) $\sqrt{-20}$

$= \sqrt{(-1)(20)}$

$= \sqrt{-1}\sqrt{20}$

$= 2i\sqrt{5}$

9. Complete the table, given that $i = \sqrt{-1}$ and $i^2 = -1$.

i	$= i^1 = i$
$i \times i$	$= i^2 = -1$
$i \times i \times i$	$= i^3 = -i$
$i \times i \times i \times i$	$= i^4 = 1$
$i \times i \times i \times i \times i$	$= i^5 = i$
$i \times i \times i \times i \times i \times i$	$= i^6 = -1$

Describe the pattern formed from this sequence.

What strategy could be used to simplify $i^n, n \in N$? [e.g., i^{32}.]

$i^{32} = (i^4)^8 = 1^8 = 1$

10. Simplify each of the following: HW

(i) i^{30} (ii) i^{11} (iii) i^{19}

(iv) i^{21} (v) i^{-4}

11. Simplify the following: HW

(i) $i^{16} + i^{10} + i^{6} - i^{12}$

(ii) $i^{3} - i^{11} + i^{17} - i^{29}$

Example 1

Solve the equation $x^2 + 25 = 0$.

$$x^2 = -25$$

$$x = \sqrt{-25}$$

$$x = 5i$$

$$i = \sqrt{-1}$$
$$\Rightarrow i^2 = -1$$

2. Solve each of the following equations, giving your answer in the form bi, where b is a real number.

(i) $x^2 + 9 = 0$ (ii) $x^2 + 12 = 0$ HW

Example 2

Solve the equation $x^2 + 2x + 2 = 0$.

$a=1 \quad b=2 \quad c=2$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

note: the 2 imaginary solutions are conjugates

7. Solve each of the following equations using the quadratic formula and give your answer in the form $a + bi, a, b \in R$:

(i) $x^2 - 2x + 17 = 0$ (ii) $x^2 - 4x + 13 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

HW

7. Solve each of the following equations using the quadratic formula and give your answer in the form $a + bi, a, b \in R$:

(iii) $x^2 - 10x + 26 = 0$ (iv) $x^2 - 8x + 52 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

HW

8. Solve the equation $2z^2 - 8z + 9 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

HW

Example 3

If $z_1 = 2 + 3i$, $z_2 = 3 - 4i$ and $z_3 = 1 + 5i$, express each of the following complex numbers in the form $a + bi$.

(i) $z_1 + z_3$ (ii) $z_2.z_3$ (iii) $z_1(z_2 + z_3)$