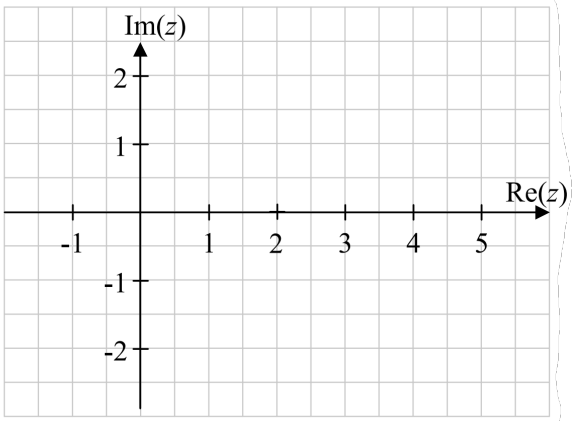
Question 2 **(25 marks)**

Let $z_1 = 1 - 2i$, where $i^2 = -1$.

(a) The complex number z_1 is a root of the equation $2z^3 - 7z^2 + 16z - 15 = 0$.
Find the other two roots of the equation.

(b) **(i)** Let $w = z_1\overline{z}_1$, where $\overline{z}_1$ is the conjugate of z_1. Plot z_1, $\overline{z}_1$ and w on the Argand diagram and label each point.



(ii) Find the measure of the acute angle, $\overline{z}_1 w z_1$, formed by joining $\overline{z}_1$ to w to z_1 on the diagram above. Give your answer correct to the nearest degree.

Question 2 **(25 marks)**

Let $z_1 = 1 - 2i,$ where $i^2 = -1.$

(a) The complex number z_1 is a root of the equation $2z^3 - 7z^2 + 16z - 15 = 0.$
Find the other two roots of the equation.

$z_1 = 1 - 2i$ a root $\Rightarrow \overline{z}_1 = 1 + 2i$ a root.

$(z - 1 + 2i)(z - 1 - 2i) = z^2 - 2z + 5,$ a factor

Hence, $(z^2 - 2z + 5)(az + b) = 2z^3 - 7z^2 + 16z - 15$

Equate coefficients: $a = 2$ and $b - 2a = -7 \quad \Rightarrow \quad b = -3$

Third factor: $2z - 3 \quad \Rightarrow \quad z = \frac{3}{2}$

Or

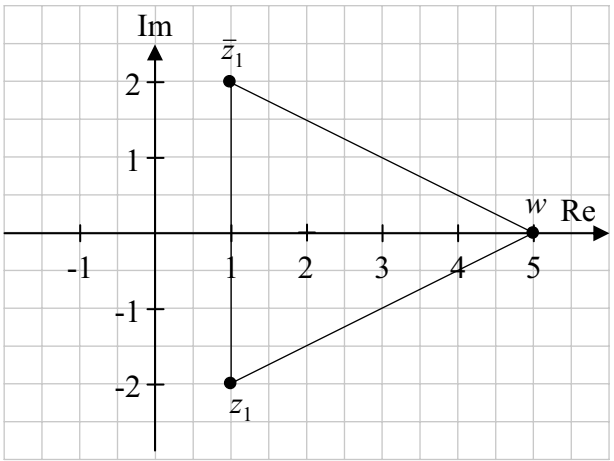
$(2z^3 - 7z^2 + 16z - 15) \div (z^2 - 2z + 5) = 2z - 3$

Third factor: $2z - 3 \quad \Rightarrow \quad z = \frac{3}{2}$

Other roots: $z_2 = 1 + 2i, z_3 = \frac{3}{2}$

(b) **(i)** Let $w = z_1.\overline{z}_1,$ where $\overline{z}_1$ is the conjugate of z_1. Plot z_1, $\overline{z}_1$ and w on the Argand diagram and label each point.

$w = (1 - 2i)(1 + 2i)$
$= 5$

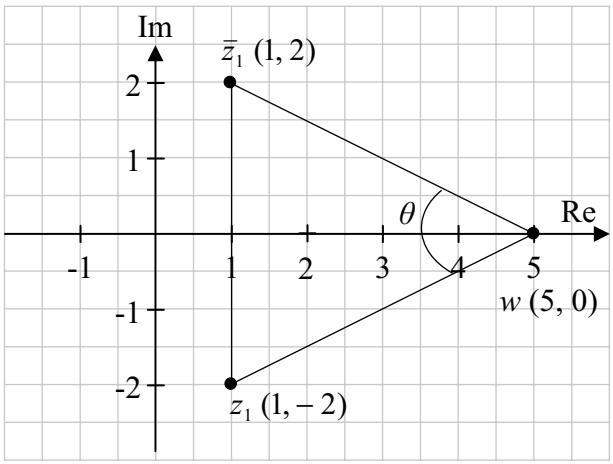


[7]

(ii) Find the measure of the acute angle, $\overline{z}_1 w z_1$, formed by joining $\overline{z}_1$ to w to z_1 on the diagram above. Give your answer correct to the nearest degree.

$\tan \frac{1}{2} \angle \overline{z}_1 w z_1 = \frac{2}{4} \quad \Rightarrow \quad \frac{1}{2} | \angle \overline{z}_1 w z_1 | = 26 \cdot 57 \quad \Rightarrow \quad | \angle \overline{z}_1 w z_1 | = 53 \cdot 14 \approx 53°$

OR



$| z_1 w | = \sqrt{(0+2)^2 + (5-1)^2} = \sqrt{16+4} = \sqrt{20}$

$| z_1 w | = \sqrt{20} \qquad | \overline{z}_1 w | = \sqrt{20} \qquad | \overline{z}_1 z_1 | = 4$

Cosine rule:

$$4^2 = (\sqrt{20})^2 + (\sqrt{20})^2 - 2(\sqrt{20})(\sqrt{20}) \cos\theta$$

$$40 \cos\theta = 24$$

$$\cos\theta = \frac{24}{40} = 0 \cdot 6$$

$$| \theta | = 53 \cdot 13 \approx 53°$$

[8]

Question 2

(a) Scale 5D (0, 2, 3, 4, 5)

Low Partial Credit:

- Identifies another root
- Forms an equation

Mid Partial Credit:

- Works with correct quadratic factor
- Indicates division of quadratic into cubic

High Partial Credit:

- Finds third factor

(b)(i) Scale 10C (0, 5, 7, 10)

Low Partial Credit:

- Plots one point correctly
- Finds $\bar{z}_1$

High Partial Credit:

- Points plotted but not labelled or labelled incorrectly
- Two points plotted and labelled
- Calculates w

(b)(ii) Scale 10C (0, 5, 7, 10)

Low Partial Credit:

- Length of any one side of triangle calculated correctly
- Correct definition of trig ratio
- Correct $\cos$ rule
- Recognises the half-angle

High Partial Credit:

- $\cos$ value calculated but angle not found
- $\tan$ value of half-angle calculated