Question 4 **(25 marks)**

(a) Differentiate the function $2x^2 - 3x - 6$ with respect to x from first principles.

(b) Let $f(x) = \frac{2x}{x+2}$, $x \neq -2$, $x \in \mathbb{R}$. Find the co-ordinates of the points at which the slope of the tangent to the curve $y = f(x)$ is $\frac{1}{4}$.

Question 4 **(25 marks)**

(a) Differentiate the function $2x^2 - 3x - 6$ with respect to x from first principles.

$$f(x) = 2x^2 - 3x - 6$$
$$f(x+h) = 2(x+h)^2 - 3(x+h) - 6 = 2x^2 + 4xh + 2h^2 - 3x - 3h - 6$$
$$f(x+h) - f(x) = 4xh + 2h^2 - 3h$$
$$\underset{h \to 0}{Limit}\left(\frac{f(x+h) - f(x)}{h}\right) = \underset{h \to 0}{Limit}\left(\frac{4xh + 2h^2 - 3h}{h}\right) = 4x - 3$$

(b) Let $f(x) = \dfrac{2x}{x+2}$, $x \neq -2$, $x \in \mathbb{R}$. Find the co-ordinates of the points at which the slope of the tangent to the curve $y = f(x)$ is $\dfrac{1}{4}$.

$$f(x) = \frac{2x}{x+2}$$
Let $u(x) = 2x \Rightarrow u'(x) = 2$ and $v(x) = x + 2 \Rightarrow v'(x) = 1$
$$f'(x) = \frac{(x+2)(2) - 2x(1)}{(x+2)^2} = \frac{4}{(x+2)^2}$$
$$f'(x) = \frac{1}{4} \Rightarrow \frac{4}{(x+2)^2} = \frac{1}{4}$$
$$\Rightarrow 16 = (x+2)^2$$

$\Rightarrow x + 2 = 4$ or $x + 2 = -4$
$\Rightarrow x = 2$ or $x = -6$

or

$x^2 + 4x - 12 = 0$
$(x-2)(x+6) = 0$
$\Rightarrow x - 2 = 0$ or $x + 6 = -0$
$\Rightarrow x = 2$ or $x = -6$

$$f(-6) = \frac{-12}{-6+2} = 3 \text{ and } f(2) = \frac{4}{2+2} = 1$$
Points $(-6, 3)$ and $(2, 1)$

Question 4

(a) Scale 15D (0, 5, 9, 12, 15)

Low Partial Credit:
- Introduces $f(x+h)$

Mid Partial Credit:
- $f(x+h)-f(x)$ expressed (need not be simplified)
- RHS only

High Partial Credit:
- $\underset{h\to 0}{Limit}\dfrac{f(x+h)-f(x)}{h}$ (need not be simplified)

(b) Scale 10D (0, 3, 7, 8, 10)

Low Partial Credit:
- Either $\dfrac{du}{dx}$ or $\dfrac{dv}{dx}$ correct
- No differentiation but writes $f'(x)=\dfrac{1}{4}$

Mid Partial Credit:
- $f'(x)$ correct but not simplified

High Partial Credit:
- Correct values of x from students work