Question 5 **(25 marks)**

(a) Find $\int 5\cos 3x\,dx$.

(b) The slope of the tangent to a curve $y = f(x)$ at each point (x, y) is $2x - 2$.
The curve cuts the x-axis at $(-2, 0)$.

(i) Find the equation of $f(x)$.

(ii) Find the average value of f over the interval $0 \le x \le 3, x \in \mathbb{R}$.

Question 5 **(25 marks)**

(a) Find $\int 5\cos 3x\,dx$.

$$\int 5\cos 3x\,dx = \tfrac{5}{3}\sin 3x + c$$

(b) The slope of the tangent to a curve $y = f(x)$ at each point (x, y) is $2x - 2$.
The curve cuts the x-axis at $(-2, 0)$.

(i) Find the equation of $f(x)$.

$$\int dy = \int (2x - 2)dx$$
$$\Rightarrow y = x^2 - 2x + c$$
At $x = -2, y = 0 \quad \Rightarrow \quad 0 = 4 + 4 + c \quad \Rightarrow \quad c = -8$
Hence, $y = x^2 - 2x - 8$

(ii) Find the average value of f over the interval $0 \le x \le 3, x \in \mathbb{R}$.

Average value: $\dfrac{1}{b-a}\displaystyle\int_a^b f(x)dx$

$$\frac{1}{3-0}\int_0^3 \left(x^2 - 2x - 8\right)dx = \frac{1}{3}\left[\frac{x^3}{3} - x^2 - 8x\right]_0^3$$
$$= \frac{1}{3}\left[\frac{27}{3} - 9 - 24\right] = -8$$

Question 5

(a) Scale 5B (0, 3, 5)
Partial Credit:
- Some correct integration
- Integrand does not contain c
- c only

(b)(i) Scale 10C (0, 5, 7, 10)
Low Partial Credit:
- Some correct integration
- Integrand does not contain c
- c only
- $\frac{dy}{dx} = 2x - 2$ or $\frac{dy}{dx} =$ slope of tangent

High Partial Credit:
- Substitutes $(-2, 0)$ but c not simplified

Note: must have 'c' in equation to get high partial marks

(b)(ii) Scale 10C (0, 5, 7, 10)
Low Partial Credit:
- Correct formula only
- Some correct integration
- Indication of integration with correct limits
- If only values used e.g. $f(0)$, $f(1)$, $f(2)$ etc. when $0 \le x \le 3$, give Low Partial Credit for two or more values

High Partial Credit:
- Limits inserted into function but not calculated
- $\frac{1}{(b-a)}$ missing from formula

Note: NO CREDIT – differentiation
NO CREDIT – no integration