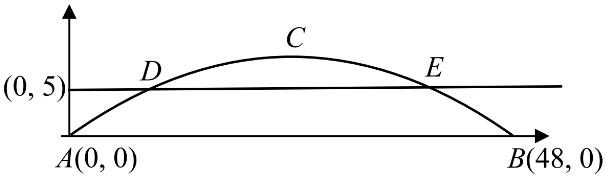
Question 8 **(50 marks)**

In 2011, a new footbridge was opened at Mizen Head, the most south-westerly point of Ireland.

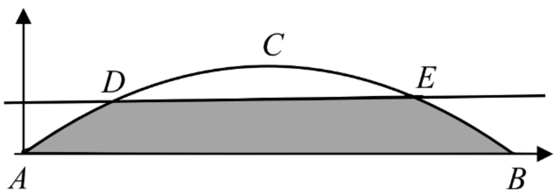
The arch of the bridge is in the shape of a parabola, as shown. The length of the span of the arch, $[AB]$, is 48 metres.



(a) Using the co-ordinate plane, with $A(0, 0)$ and $B(48, 0)$, the equation of the parabola is $y = -0\cdot013x^2 + 0\cdot624x$. Find the co-ordinates of C, the highest point of the arch.

(b) The perpendicular distance between the walking deck, $[DE]$, and $[AB]$ is 5 metres. Find the co-ordinates of D and of E. Give your answers correct to the nearest whole number.

(c) Using integration,
Give your answer



(d) Write the equation of the parabola in part **(a)** in the form $y - k = p(x - h)^2$, where k, p, and h are constants.

(e) Using what you learned in part **(d)** above, or otherwise, write down the equation of a parabola for which the coefficient of x^2 is -2 and the co-ordinates of the maximum point are $(3, -4)$.

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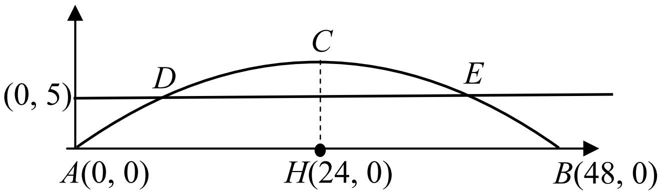


(a) Using the co-ordinate plane, with $A(0, 0)$ and $B(48, 0)$, the equation of the parabola is $y = -0\cdot013x^2 + 0\cdot624x$. Find the co-ordinates of C, the highest point of the arch.

$$y = -0\cdot013x^2 + 0\cdot624x$$

$$\Rightarrow \frac{dy}{dx} = -0\cdot026x + 0\cdot624 = 0 \Rightarrow x = 24$$

$$y = -0\cdot013x^2 + 0\cdot624x = -0\cdot013(24)^2 + 0\cdot624(24) = 7\cdot488.$$

$C(24,\ 7\cdot488)$

OR

Max height at C when x = 24

$$y = -0\cdot013x^2 + 0\cdot624x$$
$$= -0\cdot013(24)^2 + (0\cdot624)(24)$$
$$= 7\cdot488$$

$C(24,\ 7\cdot488)$

(b) The perpendicular distance between the walking deck, $[DE]$, and $[AB]$ is 5 metres.
Find the co-ordinates of D and of E. Give your answers correct to the nearest whole number.

Equation DE: $y = 5$

Equation of the parabola: $y = -0\cdot013x^2 + 0\cdot624x$.

$$5 = -0\cdot013x^2 + 0\cdot624x$$

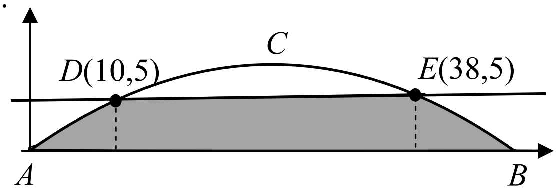
$$\Rightarrow 0\cdot013x^2 - 0\cdot624x + 5 = 0$$

$$x = \frac{0\cdot624 \pm \sqrt{0\cdot624^2 - 4(0\cdot013)5}}{2(0\cdot013)} = \frac{0\cdot624 \pm 0\cdot360}{0\cdot026}$$

$$x = 37\cdot8 \text{ or } x = 10\cdot15$$

$D(10, 5)$, $E(38, 5)$

(c) Using integration, find the area of the shaded region, *ABED,* shown in the diagram below. Give your answer correct to the nearest whole number.



$$\text{Area } ABED = \int_0^{10} y\,dx + \text{Area of rectangle} + \int_{38}^{48} y\,dx$$

$$= 2\int_0^{10} y\,dx + (38-10)\times 5$$

$$= 2\int_0^{10} (-0{\cdot}013x^2 + 0{\cdot}624x)dx + 140$$

$$= 2\left[\frac{-0{\cdot}013x^3}{3} + \frac{0{\cdot}624x^2}{2}\right]_0^{10} + 140$$

$$= 2\left[-\frac{0{\cdot}013(10)^3}{3} + \frac{0{\cdot}624(10)^2}{2} - 0\right] + 140$$

$$= 2\left[-\frac{13}{3} + 31{\cdot}2\right] + 140$$

$$= 193{\cdot}7$$

$$\approx 194 \text{ m}^2$$

OR

Contd…

Area under curve between A and B:

$$= \int_0^{48} (-0 \cdot 013x^2 + 0 \cdot 624x)dx$$

$$= \left[\frac{-0 \cdot 013x^3}{3} + \frac{0 \cdot 624x^2}{2} \right]_0^{48}$$

$$= \left[-\frac{0 \cdot 013(48)^3}{3} + \frac{0 \cdot 624(48)^2}{2} - 0 \right]$$

$$= 239 \cdot 616$$

Translate curve vertically downwards and find area under the curve between *D* and *E*:

$$= \int_{10}^{38} (-0 \cdot 013x^2 + 0 \cdot 624x - 5)dx$$

$$= \left[\frac{-0 \cdot 013x^3}{3} + \frac{0 \cdot 624x^2}{2} - 5x \right]_{10}^{38}$$

$$= \left[-\frac{0 \cdot 013(38)^3}{3} + \frac{0 \cdot 624(38)^2}{2} - 5(38) \right] - \left[\frac{-0 \cdot 013(10)^3}{3} + \frac{0 \cdot 624(10)^2}{2} - 5(10) \right]$$

$$= 22 \cdot 7493 + 23 \cdot 133$$

$$= 45 \cdot 8826$$

Shaded area $= 239 \cdot 616 - 45 \cdot 8826$

$= 193 \cdot 73$

$\approx 194 \text{ m}^2$

(d) Write the equation of the parabola in part **(a)** in the form $y-k=p(x-h)^2$, where k, p, and h are constants.

$$
\begin{aligned}
y &= -0\cdot013x^2 + 0\cdot624x \\
&= -0\cdot013\left(x^2 - 48x\right) \\
&= -0\cdot013\left(x^2 - 48x + (-24)^2 - (-24)^2\right) \\
&= -0\cdot013(x-24)^2 + 7\cdot488 \\
\Rightarrow y - 7\cdot488 &= -0\cdot013(x-24)^2
\end{aligned}
$$

(e) Using what you learned in part **(d)** above, or otherwise, write down the equation of a parabola for which the coefficient of x^2 is −2 and the co-ordinates of the maximum point are $(3,-4)$.

Given function: coefficient of x^2, $-0\cdot013$; maximum point $(24, 7\cdot488)$

New function: coefficient of x^2, -2; maximum point $(3,-4)$

Function: $y+4=-2(x-3)^2$

OR

Given function: $y-7\cdot488=-0\cdot013(x-24)^2$

$$y-(\text{max height})=(\text{coefficient of } x^2)(x-x_{\text{max}})^2$$

New parabola: max height: -4

coefficient of x^2: -2

x_{max} 3

$$
\begin{aligned}
y-(-4) &= -2(x-3)^2 \\
y+4 &= -2(x-3)^2
\end{aligned}
$$

Question 8

(a) Scale 15C (0, 7, 10, 15)

Low Partial Credit:
- Identifies $x = 24$
- Any correct differentiation

High Partial Credit:
- Substitutes $x = 24$ in $f(x)$

(b) Scale 10C (0, 5, 7, 10)

Low Partial Credit:
- Recognition of line $y = 5$

High Partial Credit:
- Values in quadratic formula
- Gets x values only

Notes: - Accept $x = 10$ and $x = 38$ by trial and error from correct quadratic for full marks
- NO CREDIT - uses $x = 5$

(c) Scale 10D (0, 3, 7, 8, 10)

Low Partial Credit:
- Any area formula
- Area under curve from A to B
- Area under curve from D to E
- Correct limits

Mid Partial Credit:
- Any correct integration
- One correct area only
- Area under curve from A to B minus area under curve from D to E

High Partial Credit:
- Limits substituted but not evaluated (must state that area of rectangle is 140)
- Both areas

OR

(c) Scale 10D (0, 3, 7, 8, 10)

Low Partial Credit:
- Any one area

Mid Partial Credit:
- Translation

High Partial Credit:
- 3rd area

(d) Scale 10C (0, 5, 7, 10)

Low Partial Credit:

- Common factor of $-0{\cdot}013$ identified
- Attempt at equating like to like
- Attempt at completing square

High Partial Credit:

- Values of two of the constants found
- p correct and completion of square correct

(e) Scale 5B (0, 3, 5)

Partial Credit:

- One correct value for the equivalent of k, p or h in equation