Question 7 **(40 marks)**

(a) Three natural numbers a, b and c, such that $a^2 + b^2 = c^2$, are called a Pythagorean triple.

(i) Let $a = 2n+1$, $b = 2n^2 + 2n$ and $c = 2n^2 + 2n + 1$.
Pick one natural number n and verify that the corresponding values of a, b and c form a Pythagorean triple.

(ii) Prove that $a = 2n+1$, $b = 2n^2 + 2n$ and $c = 2n^2 + 2n + 1$, where $n \in \mathbb{N}$, will always form a Pythagorean triple.
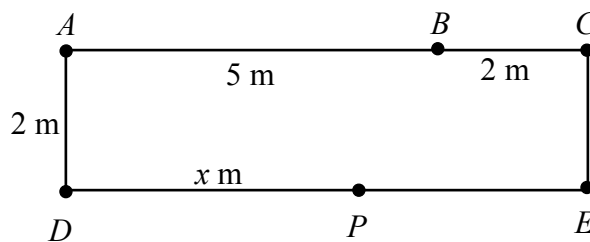
(b) *ADEC* is a rectangle with $|AC| = 7$ m
and $|AD| = 2$ m, as shown.
B is a point on $[AC]$ such that $|AB| = 5$ m.
P is a point on $[DE]$ such that $|DP| = x$ m.

A ———————— *B* — *C*

5 m   2 m

2 m

x m

D   *P*   *E*

(i) Let $f(x) = |PA|^2 + |PB|^2 + |PC|^2$.

Show that $f(x) = 3x^2 - 24x + 86$, for $0 \leq x \leq 7$, $x \in \mathbb{R}$.

(ii) The function $f(x)$ has a minimum value at $x = k$.

Find the value of k and the minimum value of $f(x)$.

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