Question 9 **(60 marks)**

Ciarán is preparing food for his baby and must use cooled boiled water. The equation $y = Ae^{kt}$ describes how the boiled water cools. In this equation:

- t is the time, in minutes, from when the water boiled,
- y is the *difference* between the water temperature and room temperature at time t, measured in degrees Celsius,
- A and k are constants.

The temperature of the water when it boils is $100°\text{C}$ and the room temperature is a constant $23°\text{C}$.

(a) Write down the value of the temperature difference, y, when the water boils, and find the value of A.

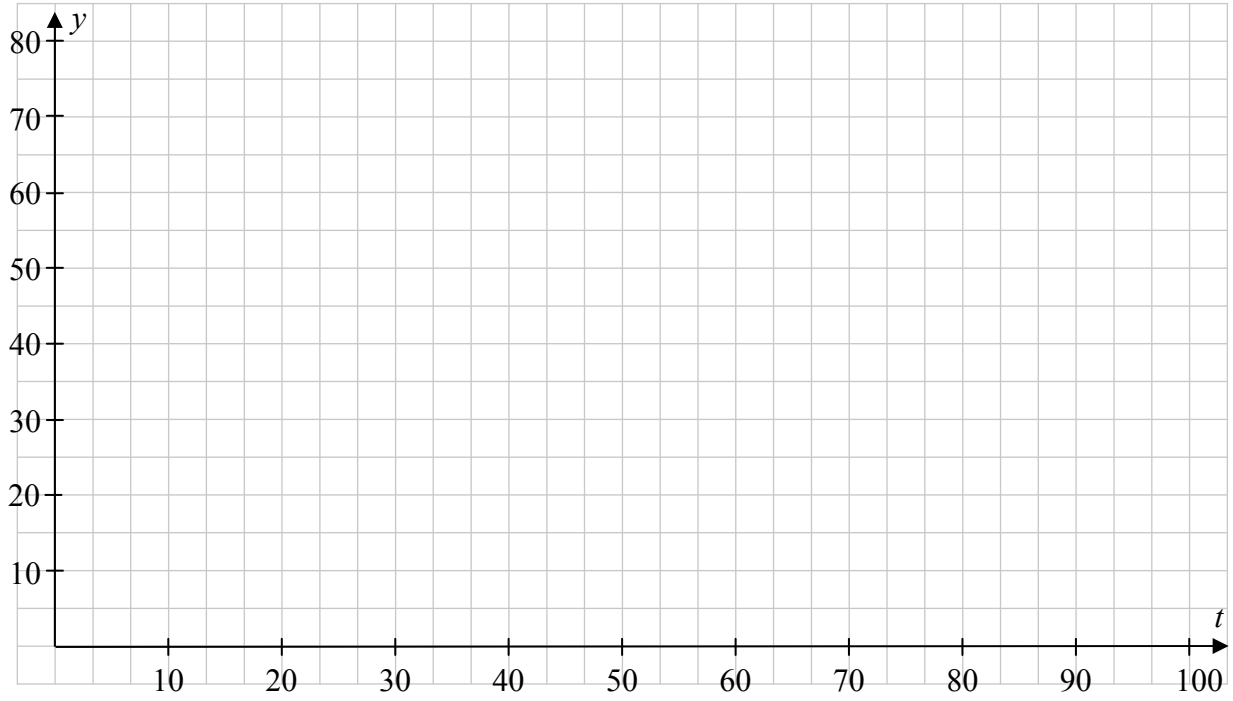
$y =$

$A =$

(b) After five minutes, the temperature of the water is $88°\text{C}$.
Find the value of k, correct to three significant figures.

(c) Ciarán prepares the food for his baby when the water has cooled to $50°\text{C}$. How long does it take, correct to the nearest minute, for the water to cool to this temperature?

(d) Using your values for A and k, sketch the curve $f(t) = Ae^{kt}$ for $0 \le t \le 100,\ t \in \mathbb{R}$.



(e) **(i)** On the same diagram, sketch a curve $g(t) = Ae^{mt}$, showing the water cooling at a *faster* rate, where A is the value from part **(a)**, and m is a constant. Label each graph clearly.

(ii) Suggest one possible value for m for the sketch you have drawn and give a reason for your choice.

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(f) **(i)** Find the rates of change of the function $f(t)$ after 1 minute and after 10 minutes. Give your answers correct to two decimal places.

(ii) Show that the rate of change of $f(t)$ will always increase over time.