## Question 2

(a) Prove that $\cos 2 A=\cos ^{2} A-\sin ^{2} A$.
(b) The diagram shows part of the circular end of a running track with three running lanes shown. The centre of each of the circular boundaries of the lanes is at $O$.

Kate runs in the middle of lane 1, from $A$ to $B$ as shown.

Helen runs in the middle of lane 2, from $C$ to $D$ as shown.

Helen runs 3 m further than Kate.
$|\angle A O B|=|\angle C O D|=\theta$ radians.
If each lane is 1.2 m wide, find $\theta$.

(a) Prove that $\cos 2 A=\cos ^{2} A-\sin ^{2} A$.

$$
\begin{aligned}
& \cos (A+B)=\cos A \cos B-\sin A \sin B \\
& \cos 2 A=\cos (A+A)=\cos A \cos A-\sin A \sin A=\cos ^{2} A-\sin ^{2} A
\end{aligned}
$$

(b) The diagram shows part of the circular end of a running track with three running lanes shown. The centre of each of the circular boundaries of the lanes is at $O$.

Kate runs in the middle of lane 1, from $A$ to $B$ as shown.

Helen runs in the middle of lane 2, from $C$ to $D$ as shown.

Helen runs 3 m further than Kate.
$|\angle A O B|=|\angle C O D|=\theta$ radians.
If each lane is 1.2 m wide, find $\theta$.


Kate: $|A B|=s_{1}=|O A| \theta=r \theta$
Helen: $|C D|=s_{2}=(|O A|+1 \cdot 2) \theta=(r+1 \cdot 2) \theta$
$s_{1}+3=s_{2}$
$\Rightarrow r \theta+3=r \theta+1 \cdot 2 \theta$
$\Rightarrow 1 \cdot 2 \theta=3$
$\Rightarrow \theta=2 \cdot 5$ radians
(a) Scale 15C (0, 5, 10, 15)

Low Partial Credit:

- Relevant compound angle formula
- Tested with one or more values for A

High Partial Credit

- Expansion correct but not tidied
(b) Scale 10D (0, 2 ,5, 8, 10)

Low Partial Credit:

- Correct formula for finding either arc

Mid Partial Credit

- One or both arcs expressed correctly

High Partial Credit

- $\quad \theta$ not fully evaluated
- $|C D|-|A B|=3$ or equivalent statement
- Substantially correct with one non arithmetic error

