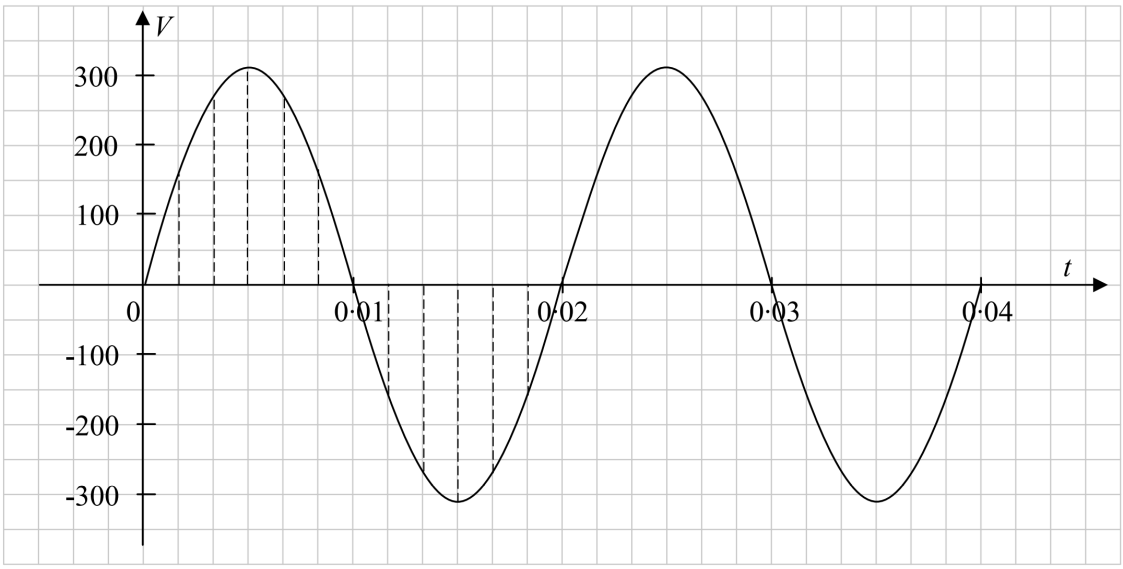
Question 4 **(25 marks)**

The graph below shows the voltage, V, in an electric circuit as a function of time, t. The voltage is given by the formula $V = 311\sin(100\pi t)$, where V is in volts and t is in seconds.



(a) **(i)** Write down the range of the function.

(ii) How many complete periods are there in one second?

(b) **(i)** The table below gives the voltage, correct to the nearest whole number, at equally spaced intervals from t_1 to t_{12} over one complete period (as shown by the dashed lines on the diagram). Use the entries given in the table and the properties of the function to complete the table.

t	t_1	t_2	t_3	t_4	t_5	$t_6 = 0{\cdot}01$	t_7	t_8	t_9	t_{10}	t_{11}	$t_{12} = 0{\cdot}02$
V	156	269	311									

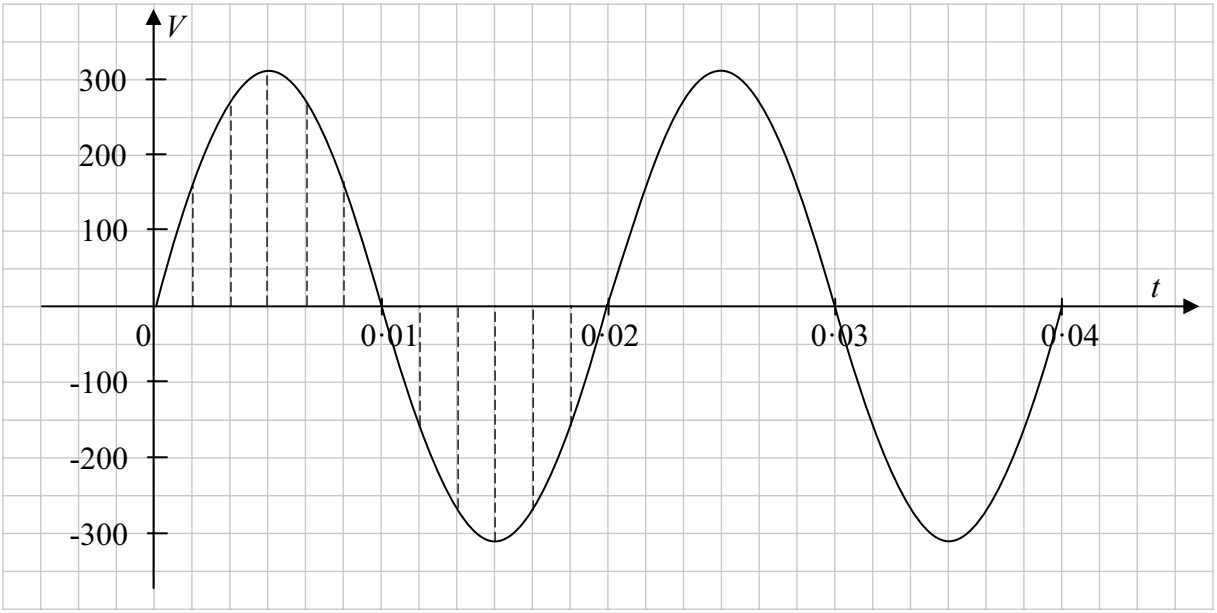
(ii) Using a calculator, or otherwise, calculate the standard deviation, σ, of the twelve values of V in the table, correct to the nearest whole number.

(c) **(i)** The standard deviation, σ, of closely spaced values of any function of the form $V = a\sin(bt)$, over 1 full period, is given by $k\sigma = V_{max}$, where k is a constant that does not depend on a or b, and V_{max} is the maximum value of the function. Use the function $V = 311\sin(100\pi t)$ to find an approximate value for k correct to three decimal places.

(ii) Using your answer in part **(c) (i)**, or otherwise, find the value of b required so that the function $V = a\sin(bt)$ has 60 complete periods in one second and the approximate value of a so that it has a standard deviation of 110 volts.

Question 4 **(25 marks)**

The graph below shows the voltage, V, in an electric circuit as a function of time, t. The voltage is given by the formula $V = 311\sin(100\pi t)$, where V is in volts and t is in seconds.



(a) **(i)** Write down the range of the function.

Range: $[-311, 311]$

(ii) How many complete periods are there in one second?

$\frac{100\pi}{2\pi} = 50$ periods per second

Or

Time for 1 period = 0·02 seconds

Number of periods in 1 second $= \frac{1}{0 \cdot 02} = 50$

(b) **(i)** The table below gives the voltage, correct to the nearest whole number, at equally spaced intervals from t_0 to t_{12} over one complete period (as shown by the dashed lines on the diagram). Use the entries given in the table and the properties of the function to complete the table.

T	t_1	t_2	t_3	t_4	t_5	$t_6 = 0{\cdot}01$	t_7	t_8	t_9	t_{10}	t_{11}	$t_{12} = 0{\cdot}02$
V	156	269	311	269	156	0	−156	−269	−311	−269	−156	0

(ii) Using a calculator, or otherwise, calculate the standard deviation, σ, of the twelve values of V in the table, correct to the nearest whole number.

$$\sigma = 219{\cdot}89 = 220$$

(c) **(i)** The standard deviation, σ, of closely spaced values of any function of the form $V = a\sin(bt)$, over 1 full period, is given by $k\sigma = V_{\max}$, where k is a constant that does not depend on a or b, and $V_{\max}$ is the maximum value of the function. Use the function $V = 311\sin(100\pi t)$ to find an approximate value for k correct to three decimal places.

$$k = \frac{V_{\max}}{\sigma} = \frac{311}{220} \approx 1{\cdot}414$$

(ii) Using your answer in part **(c) (i)**, or otherwise, find the value of b required so that the function $V = a\sin(bt)$ has 60 complete periods in one second and the approximate value of a so that it has a standard deviation of 110 volts.

$$\frac{b}{2\pi} = 60 \Rightarrow b = 120\pi = 377$$

$$k\sigma = V_{\max} \quad \Rightarrow \quad V_{\max} = 1{\cdot}414 \times 110 = 155{\cdot}54 \Rightarrow a = 156$$

Question 4 **(25 marks)**

(a)(i) (ii) Scale 10C (0, 3, 7, 10)

Low Partial Credit:

- Some reference to 311 or -311
- Some indication that term' range' understood
- Ranges (other than correct one) between $\pm 300, \pm 320$ (consistent)
- Some reference to how long it takes to complete a cycle
- Time for one period found

High Partial Credit:

- Correct range
- Correct number of periods

(b)(i) (ii) Scale 10C (0, 3, 7, 10)

Low Partial Credit:

- At least three further entries in table correct
- Formula for standard deviation

High Partial Credit:

- Table correct
- Errors in table (with at least three additional entries) but standard deviation correct from candidates work

(c)(i) (ii) Scale 5C (0, 2, 3, 5)

Low Partial Credit:

- k isolated in formula
- Value(s) entered in formula
- $\frac{b}{2\pi}$ written

High Partial Credit:

- k or b or a found