## Question 5

(25 marks)
The line $R S$ cuts the $x$-axis at the point $R$ and the $y$-axis at the point $S(0,10)$, as shown. The area of the triangle $R O S$, where $O$ is the origin, is $\frac{125}{3}$.
(a) Find the co-ordinates of $R$.
(b) Show that the point $E(-5,4)$ is on the line $R S$.

(c) A second line $y=m x+c$, where $m$ and $c$ are positive constants, passes through the point $E$ and again makes a triangle of area $\frac{125}{3}$ with the axes. Find the value of $m$ and the value of $c$.

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(a) Find the co-ordinates of $R$.

$$
\begin{aligned}
& \text { Area } R O S=\frac{1}{2}|R O| \cdot|O S|=\frac{125}{3} \\
& \Rightarrow \frac{1}{2}|R O|(10)=\frac{125}{3} \\
& \Rightarrow|R O|=\frac{25}{3} \\
& R\left(-\frac{25}{3}, 0\right)
\end{aligned}
$$


(b) Show that the point $E(-5,4)$ is on the line $R S$.

Slope RS $=\frac{10-0}{0+\frac{25}{3}}=\frac{6}{5} \quad$ Slope $\mathrm{ES}=\frac{10-4}{0+5}=\frac{6}{5} \quad$ Slope $\mathrm{ER}=\frac{4-0}{-5+\frac{25}{3}}=\frac{6}{5}$
Any two slopes correct $\Rightarrow(-5,4) \in R S$
Or
RS: $y-10=\frac{6}{5}(x-0) \Rightarrow 6 x-5 y+50=0$
$6(-5)-5(4)+50=-30-20+50=0 \Rightarrow(-5,4) \in R S$
(c) A second line $y=m x+c$, where $m$ and $c$ are positive constants, passes through the point $E$ and again makes a triangle of area $\frac{125}{3}$ with the axes. Find the value of $m$ and the value of $c$.
$y=m x+c$ cuts x -axis at $P\left(-\frac{c}{m}, 0\right)$ and cuts y -axis at $\mathrm{Q}(0, \mathrm{c})$
Area $\triangle P O Q=\frac{1}{2}\left|0-\left(-\frac{c}{m}\right) c\right|=\frac{1}{2}\left|\frac{c^{2}}{m}\right|=\frac{125}{3} \Rightarrow m=\frac{3 c^{2}}{250}$
$(-5,4) \in y=m x+c \Rightarrow 4=-5 m+c \Rightarrow 4=-5\left(\frac{3 c^{2}}{250}\right)+c \Rightarrow 3 c^{2}-50 c+200=0$
$\Rightarrow(3 c-20)(c-10)=0 \Rightarrow c=\frac{20}{3}$ or $c=10$
$c=\frac{20}{3}$
Hence, $m=\frac{3 c^{2}}{250}=\frac{3\left(\frac{20}{3}\right)^{2}}{250}=\frac{400}{750}=\frac{8}{15}$
(a) Scale 10C (0, 3, 7, 10)

Low Partial Credit:
Relevant area of triangle formula
High Partial Credit:

- $\quad|O R|$ found but $x$ ordinate of $R$ not stated
- Substantially correct work with one error
(b) Scale 10C (0, 3, 7, 10)

Low Partial Credit:

- Effort at finding one slope
- Effort at finding equation of $R S$

High Partial Credit:

- Relevant conclusion not stated or implied
- $E$ inserted into equation of $R S$ but relevant conclusion not stated or implied
(c) $\operatorname{Scale} 5 \mathrm{C}(0,2,3,5)$

Low Partial Credit:

- Effort at finding intercept on one or both axes
- Effort at inserting $(-5,4)$ into $y=m x+c$

High Partial Credit:

- Either $c$ or $m$ found

