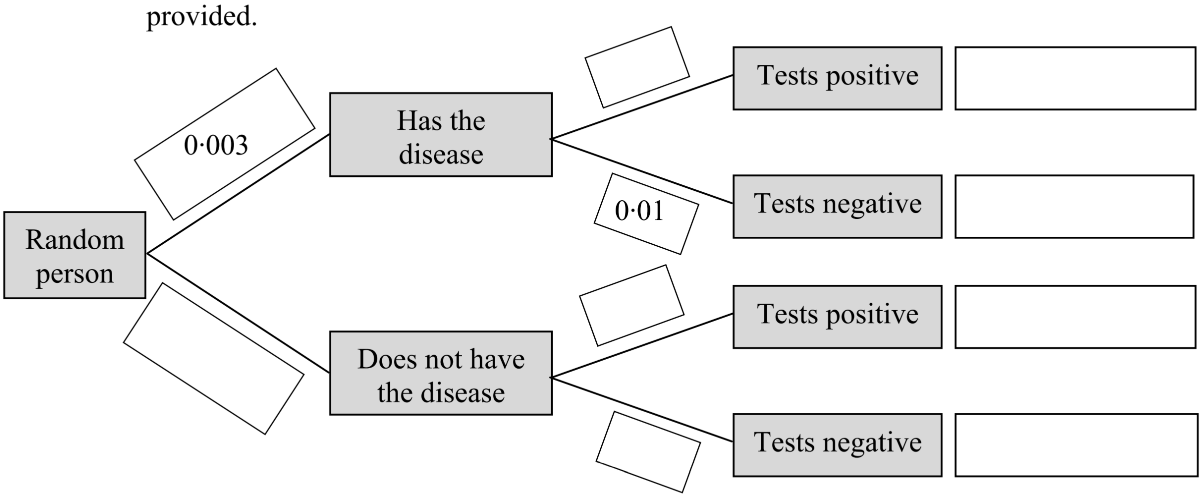
Question 8 **(45 marks)**

Blood tests are sometimes used to indicate if a person has a particular disease. Sometimes such tests give an incorrect result, either indicating the person has the disease when they do not (called a false positive) or indicating that they do not have the disease when they do (called a false negative).

It is estimated that 0·3% of a large population have a particular disease. A test developed to detect the disease gives a false positive in 4% of tests and a false negative in 1% of tests. A person picked at random is tested for the disease.

(a) **(i)** Write the probability associated with each branch of the tree diagram in the blank boxes provided.



(ii) Hence, or otherwise, calculate the probability that a person selected at random from the population tests positive for the disease.

(iii) A person tests positive for the disease. What is the probability that the person actually has the disease? Give your answer correct to three significant figures.

(iv) The health authority is considering using a test on the general population with a view to treatment of the disease. Based on your results, do you think that the above test would be an effective way to do this? Give a reason for your answer.

(b) A generic drug used to treat a particular condition has a success rate of 51%. A company is developing two new drugs, *A* and *B*, to treat the condition. They carried out clinical trials on two groups of 500 patients suffering from the condition. The results showed that Drug *A* was successful in the case of 296 patients. The company claims that Drug *A* is more successful in treating the condition than the generic drug.

(i) Use a hypothesis test at the 5% level of significance to decide whether there is sufficient evidence to justify the company's claim. State the null hypothesis and state your conclusion clearly.

(ii) The null hypothesis was accepted for Drug *B*. Estimate the greatest number of patients in that trial who could have been successfully treated with Drug *B*.

Question 8 **(45 marks)**

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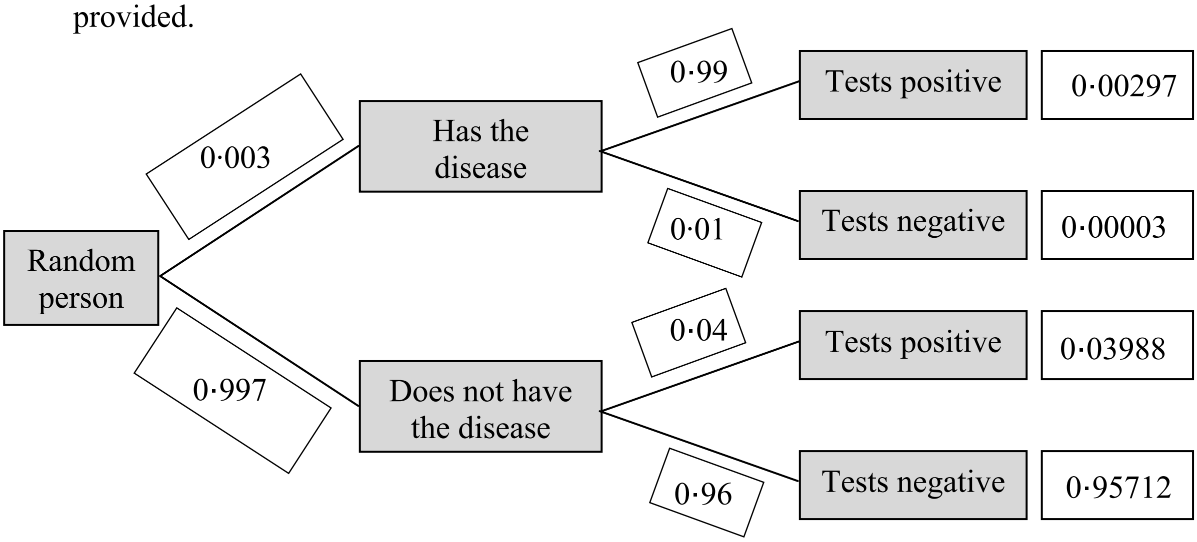

(ii) Hence, or otherwise, calculate the probability that a person selected at random from the population tests positive for the disease.

$$\text{P(Positive test)} = 0\cdot 00297 + 0\cdot 03988 = 0\cdot 04285$$

(iii) A person tests positive for the disease. What is the probability that the person actually has the disease. Give your answer correct to three significant figures.

$$\text{P(Has disease}|\text{positive test)} = \frac{0\cdot 00297}{0\cdot 04285} = 0\cdot 0693$$

(iv) The health authority is considering using a test on the general population with a view to treatment of the disease. Based on your results, do you think that the above test would be an effective way to do this? Give a reason for your answer.

Test is not very useful.
A person who tests positive has the disease only 7% of the time.

(b) A generic drug used to treat a particular condition has a success rate of 51%. A company is developing two new drugs, A and B, to treat the condition. They carried out clinical trials on two groups of 500 patients suffering from the condition. The results showed that Drug A was successful in the case of 296 patients. The company claims that Drug A is more successful in treating the condition than the generic drug.

(i) Use a hypothesis test at the 5% level of significance to decide whether there is sufficient evidence to justify the company's claim. State the null hypothesis and state your conclusion clearly.

H_0: The new drug is not more successful than the generic drug.
$p = 0 \cdot 51$

95% margin of error $= \dfrac{1}{\sqrt{500}} = 0 \cdot 045$

The success rate for the new drug is $\dfrac{296}{500} = 0 \cdot 592$.

This is outside the interval $[0 \cdot 51 - 0 \cdot 045,\ 0 \cdot 51 + 0 \cdot 045] = [0 \cdot 465,\ 0 \cdot 555]$

Result is significant, reject the null hypothesis.

There is evidence to conclude that the new drug is more successful than the generic.

Or

H_0: The new drug is not more successful than the generic drug.
H_1: The new drug is more successful than the generic drug.
$p = 0 \cdot 51$

95% margin of error $= \dfrac{1}{\sqrt{500}} = 0 \cdot 045$

The success rate for the new drug is $\dfrac{296}{500} = 0 \cdot 592$.

The 95% confidence interval for the population is
$0 \cdot 592 - 0 \cdot 045 < p < 0 \cdot 592 + 0 \cdot 045 = 0 \cdot 547 < p < 0 \cdot 637$

$p = 0 \cdot 51$ is outside this interval.

Result is significant, reject the null hypothesis.

There is evidence to conclude that the new drug is more successful than the generic

(ii) The null hypothesis was accepted for Drug *B*. Estimate the greatest number of patients in that trial who could have been successfully treated with Drug *B*.

The result must lie in the interval $[0\cdot 465, 0\cdot 555\,]$

Thus, $\dfrac{n}{500} < 0\cdot 555 \Rightarrow n < 277\cdot 5$

Hence, 277 patients.

Or

$k - 0\cdot 045 < 0\cdot 51 < k + 0\cdot 045$

$\Rightarrow k - 0\cdot 045 < 0\cdot 51$

$\Rightarrow k < 0\cdot 555$

Number of patients $< 0\cdot 555 \times 500 = 277\cdot 5$

Hence, 277 patients.

Question 8 **(45 marks)**

(a)(i) Scale 10C (0, 3, 7, 10)

Low Partial Credit:

- One element entered correctly
- One column correct
- Some indication that values lie between 0 and 1

High Partial Credit:

- Two columns correct

(ii) Scale 5C (0, 2, 3, 5)

Low Partial Credit:

- One correct value chosen
- Addition of values indicated
- Configured correctly but no values entered
- Answer outside range

High Partial Credit:

- Correct values chosen but operator incorrect

(iii) Scale 5C (0, 2, 3, 5)

Low Partial Credit:

- One or both correct value(s) chosen only
- Configured correctly but values not entered
- Answer outside range

High Partial Credit:

- Correct values chosen but incorrect operator leading to an answer within range

(iv) Scale 5B (0, 2, 5)

Partial Credit:

- Reason incorrect or incomplete

(b)(i) Scale 15D(0, 4, 7, 11, 15)

Low Partial Credit:

- One relevant step e.g. null hypothesis stated only
- Some work towards margin of error

Mid Partial Credit:

- Margin of error or observed proportion
- Margin of error and observed proportion found but fails to continue

High Partial Credit:

- Failure to state null hypothesis correctly
- Failure to contextualise answer (e.g. Stops at reject Null Hypothesis)

(b)(ii) Scale 5B (0, 2, 5)

Partial Credit:

- $\frac{n}{500}$ and stops
- Recognises interval where result must lie
- Some relevant work