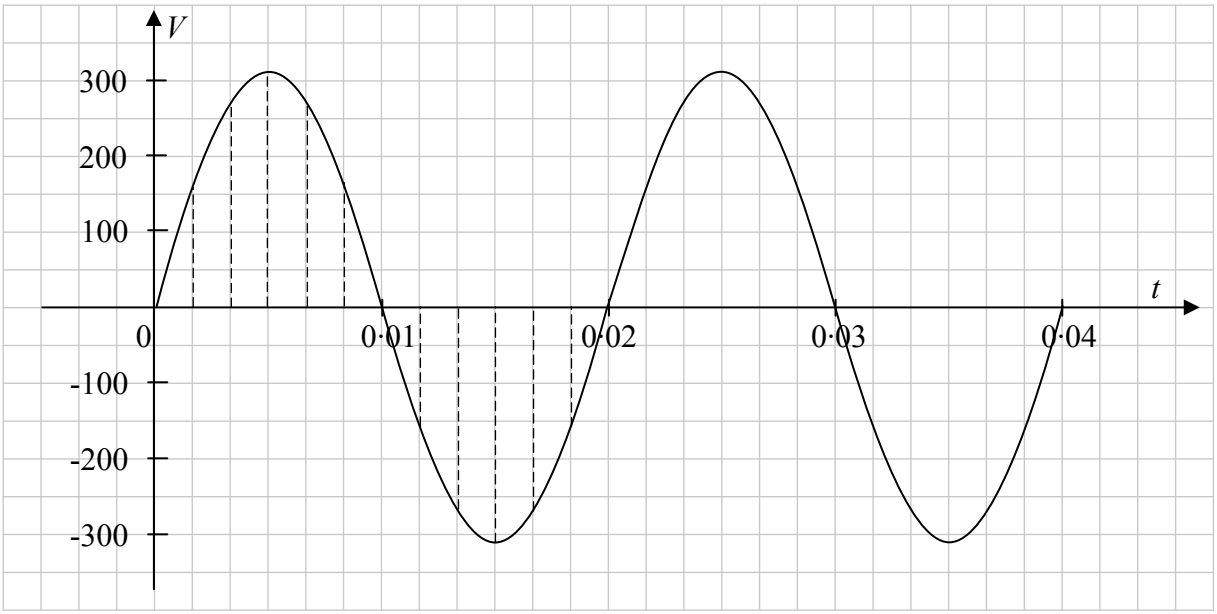
Question 4 **(25 marks)**

The graph below shows the voltage, V, in an electric circuit as a function of time, t. The voltage is given by the formula $V = 311\sin(100\pi t)$, where V is in volts and t is in seconds.



(a) **(i)** Write down the range of the function.

(ii) How many complete periods are there in one second?

(b) **(i)** The table below gives the voltage, correct to the nearest whole number, at equally spaced intervals from t_1 to t_{12} over one complete period (as shown by the dashed lines on the diagram). Use the entries given in the table and the properties of the function to complete the table.

t	t_1	t_2	t_3	t_4	t_5	$t_6 = 0{\cdot}01$	t_7	t_8	t_9	t_{10}	t_{11}	$t_{12} = 0{\cdot}02$
V	156	269	311									

(ii) Using a calculator, or otherwise, calculate the standard deviation, σ, of the twelve values of V in the table, correct to the nearest whole number.

(c) **(i)** The standard deviation, σ, of closely spaced values of any function of the form $V = a\sin(bt)$, over 1 full period, is given by $k\sigma = V_{\max}$, where k is a constant that does not depend on a or b, and $V_{\max}$ is the maximum value of the function. Use the function $V = 311\sin(100\pi t)$ to find an approximate value for k correct to three decimal places.

(ii) Using your answer in part **(c) (i)**, or otherwise, find the value of b required so that the function $V = a\sin(bt)$ has 60 complete periods in one second and the approximate value of a so that it has a standard deviation of 110 volts.