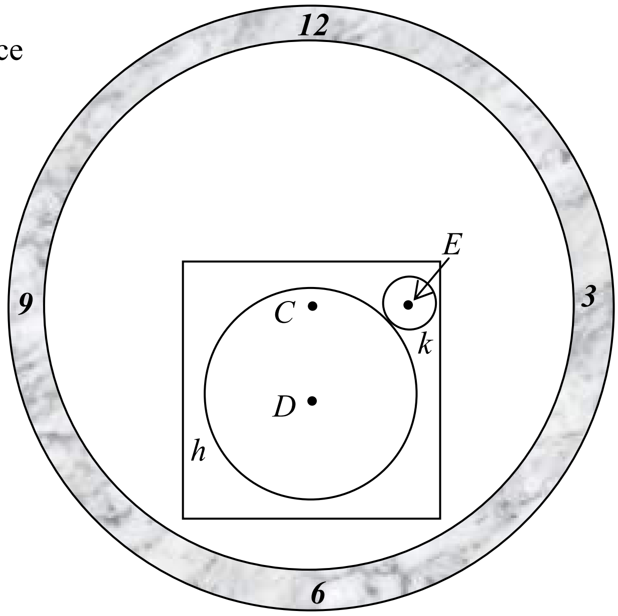
Question 9 **(60 marks)**

(a) The diagram shows a circular clock face, with the hands not shown. The square part of the clock face is glass so that the mechanism is visible. Two circular cogs, h and k, which touch externally are shown.



The point C is the centre of the clock face. The point D is the centre of the larger cog, h, and the point E is the centre of the smaller cog, k.

(i) In suitable co-ordinates, the equation of the circle h is

$$x^2 + y^2 + 4x + 6y - 19 = 0.$$

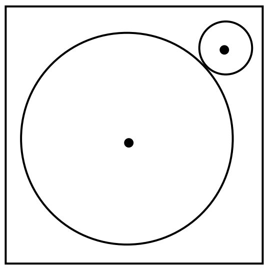
Find the radius of h, and the co-ordinates of its centre, D.

(ii) The point E has co-ordinates $(3, 2)$. Find the radius of the circle k.

(iii) Show that the distance from $C(-2, 2)$ to the line DE is half the length of $[DE]$.

(iv) The translation which maps the midpoint of $[DE]$ to the point C maps the circle k to the circle j. Find the equation of the circle j.

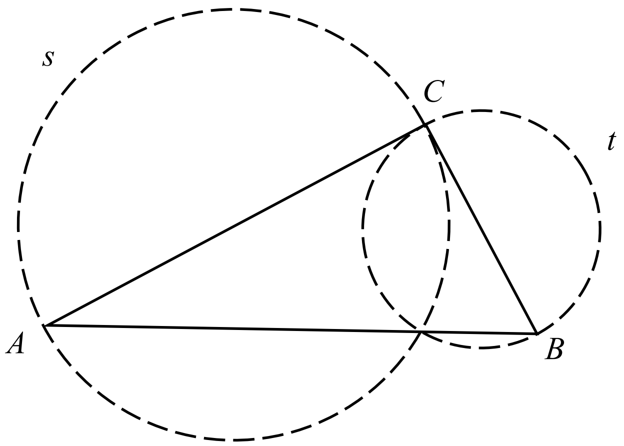
(v) The glass square is of side length l. Find the smallest whole number l such that the two cogs, h and k, are fully visible through the glass.



(b) The triangle ABC is right-angled at C.

The circle s has diameter $[AC]$ and the circle t has diameter $[CB]$.

(i) Draw the circle u which has diameter $[AB]$.

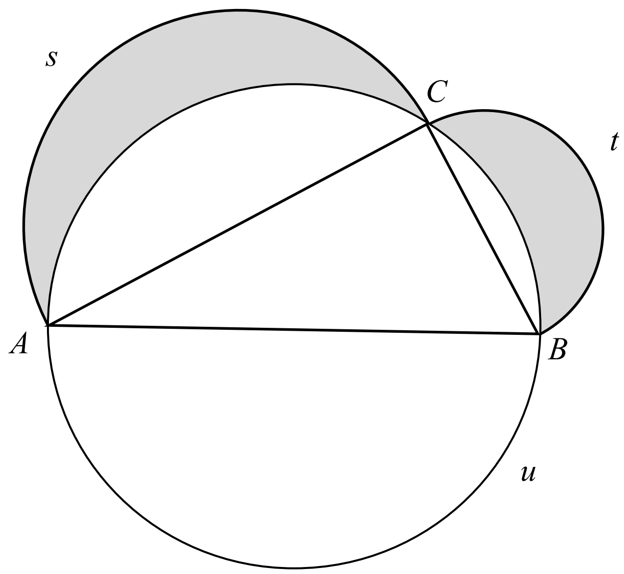


(ii) Prove that in any right-angles triangle ABC, the area of the circle u equals the sum of the areas of the circles s and t.

(iii) The diagram shows the right-angled triangle ABC and arcs of the circles s, t and u.

Each of the shaded areas in the diagram is called a lune, a crescent-shaped area bounded by arcs of the circles.

Prove that the sum of the areas of the two shaded lunes is equal to the area of the triangle ABC.



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