

Question 2**(25 marks)**

Solve the equation $x^3 - 3x^2 - 9x + 11 = 0$.

Write any irrational solution in the form $a + b\sqrt{c}$, where $a, b, c \in \mathbb{Z}$.

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(25 marks)

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$f(x) = x^3 - 3x^2 - 9x + 11$
 $f(1) = 1^3 - 3(1)^2 - 9 + 11 = 0$
 $\Rightarrow x = 1$ is a solution.
 $(x - 1)$ is a factor

$ \begin{array}{r} x^2 - 2x - 11 \\ x-1 \overline{) x^3 - 3x^2 - 9x + 11} \\ \underline{x^3 - x^2} \\ -2x^2 - 9x + 11 \\ \underline{-2x^2 + 2x} \\ -11x + 11 \\ \underline{-11x + 11} \\ 0 \end{array} $	or	$ \begin{aligned} (x-1)(x^2 + Ax - 11) &= x^3 - 3x^2 - 9x + 11 \\ \Rightarrow x^3 + Ax^2 - x - x^2 - Ax + 11 &= x^3 - 3x^2 - 9x + 11 \\ \Rightarrow A - 1 &= -3 \\ \Rightarrow A &= -2 \end{aligned} $
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or

x	x^2	$-2x$	-11
x	x^3	$-2x^2$	$-11x$
-1	$-x^2$	$2x$	11

Hence, other factor is $x^2 - 2x - 11$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)} = \frac{2 \pm \sqrt{48}}{2} = \frac{2 \pm 4\sqrt{3}}{2} = 1 \pm 2\sqrt{3}$$

Solutions: $\{1, 1 + 2\sqrt{3}, 1 - 2\sqrt{3}\}$

Scale 25E (0, 5, 10, 15, 20, 25)

Low Partial Credit:

- Effort at finding root, i.e. $f(1)$, $f(-1)$, etc.

Low

Mid Partial Credit:

- Finds one root correctly
- x^2 after division by incorrect factor
- Correct answers in decimal form from calculator with or without work

High Mid Partial Credit:

- Tries division and gets x^2 at very minimum

Note: If there is a remainder after division can only get maximum of 15 marks.

High Partial Credit:

- Having got a quadratic equation with no remainder, fills in quadratic formula • $1 \pm \sqrt{12}$

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