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$f(x) = x^3 - 3x^2 - 9x + 11$

$f(1) = 1^3 - 3(1)^2 - 9 + 11 = 0$

$\Rightarrow x = 1$ is a solution.

$(x - 1)$ is a factor

$$
\begin{array}{r}
x^2 - 2x \quad -11 \\
x-1 \overline{)\, x^3 - 3x^2 - 9x + 11} \\
x^3 - x^2 \\
\overline{-2x^2 - 9x + 11} \\
-2x^2 + 2x \\
\overline{-11x + 11} \\
-11x + 11
\end{array}
$$

or

$(x-1)(x^2 + Ax - 11) = x^3 - 3x^2 - 9x + 11$

$\Rightarrow x^3 + Ax^2 - x - x^2 - Ax + 1 = x^3 - 3x^2 - 9x + 11$

$\Rightarrow A - 1 = -3$

$\Rightarrow A = -2$

or

	x^2	$-2x$	-11
x	x^3	$-2x^2$	$-11x$
-1	$-x^2$	$2x$	11

Hence, other factor is $x^2 - 2x - 11$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)} = \frac{2 \pm \sqrt{48}}{2} = \frac{2 \pm 4\sqrt{3}}{2} = 1 \pm 2\sqrt{3}$$

Solutions: $\{1,\ 1 + 2\sqrt{3},\ 1 - 2\sqrt{3}\}$