Question 3 **(25 marks)**

Let $f(x) = -x^2 + 12x - 27$, $x \in \mathbb{R}$.

(a) **(i)** Complete Table 1 below.

Table 1							
x	3	4	5	6	7	8	9
$f(x)$	0	5			8		

(ii) Use Table 1 and the trapezoidal rule to find the approximate area of the region bounded by the graph of f and the x-axis.

(b) **(i)** Find $\int_3^9 f(x)dx$.

(ii) Use your answers above to find the percentage error in your approximation of the area, correct to one decimal place.

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$f(x)$	0	5	8	9	8	5	0

(ii) Use Table 1 and the trapezoidal rule to find the approximate area of the region bounded by the graph of f and the x-axis.

$$A = \frac{h}{2}\left[y_1 + y_n + 2\left(y_2 + y_3 + \cdots + y_{n-1}\right)\right]$$
$$= \frac{1}{2}\left[0 + 0 + 2\left(5 + 8 + 9 + 8 + 5\right)\right]$$
$$= 35 \text{ square units}$$

(a)(i) and (ii) combined Scale 15D (0, 4, 7, 11, 15)
Low Partial Credit: • Any one correct value • Writes formula
Mid Partial Credit: • Correct table
High Partial Credit: • Correct formula for trapezoidal rule, and some correct substitution with h = 1
• Completely incorrect table but applied correctly in a(ii)
• Correct table and 35 without work
Note (1): Answers in terms of h merit Mid Partial at most.
Note (2): Correct formula and some substitution gets High Partial.
Note (3): No formula and (1/5))[5 + 5 + 2(8 + 9 + 8)]= 30 gets High Partial.

(b) **(i)** Find $\int_3^9 f(x)dx$.

$$\int_3^9 \left(-x^2 + 12x - 27\right) dx$$
$$= \left[\frac{-x^3}{3} + \frac{12x^2}{2} - 27x\right]_3^9$$
$$= \left(-243 + 486 - 243\right) - \left(-9 + 54 - 81\right)$$
$$= 36$$

(ii) Use your answers above to find the percentage error in your approximation of the area, correct to one decimal place.

$$\frac{1}{36} \times 100 = 2 \cdot 8\%$$

(b)(i) and (ii) combined Scale 10C (0, 4, 8, 10)

Low Partial Credit:
• Any correct integration
• Correct substitution of f (x)
• Correct % error formula
• Correct substitution of f (x) i.e. (−x^2 +12x − 27)

High Partial Credit:
• Correct integration with some correct substitution • 97.2%

Full Credit:
• 2.8% without work for full credit.

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