Question 4 **(25 marks)**

(a) The complex numbers z_1, z_2 and z_3 are such that $\frac{2}{z_1} = \frac{1}{z_2} + \frac{1}{z_3}$, $z_2 = 2 + 3i$ and $z_3 = 3 - 2i$, where $i^2 = -1$. Write z_1 in the form $a + bi$, where $a, b \in \mathbb{Z}$.

(b) Let ω be a complex number such that $\omega^n = 1$, $\omega \neq 1$, and $S = 1 + \omega + \omega^2 + \cdots + \omega^{n-1}$. Use the formula for the sum of a finite geometric series to write the value of S in its simplest form.

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$$\frac{2}{z_1} = \frac{1}{z_2} + \frac{1}{z_3} = \frac{1}{2+3i} + \frac{1}{3-2i}$$
$$= \frac{3-2i+2+3i}{(2+3i)(3-2i)} = \frac{5+i}{12+5i}$$
$$\Rightarrow \frac{z_1}{2} = \frac{12+5i}{5+i}$$
$$= \frac{12+5i}{5+i} \times \frac{5-i}{5-i}$$
$$= \frac{65+13i}{26}$$
$$\Rightarrow z_1 = 5+i$$

(a) Scale 15D (0, 4, 7, 11, 15) – NOTE: two solutions

Low Partial Credit:
- Some rationalisation
- Some relevant rearrangement.

Mid Partial Credit:
- Gets z1 or 2/z1 in the form of (a + bi)/(c+di)

High Partial Credit:
- Correct use of conjugate in (12 + 5i)/(5+i)

or

$$\frac{1}{2+3i} = \frac{1}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{2-3i}{4+9} = \frac{2-3i}{13}$$
$$\frac{1}{3-2i} = \frac{1}{3-2i} \cdot \frac{3+2i}{3+2i} = \frac{3+2i}{4+9} = \frac{3+2i}{13}$$
$$\frac{1}{2+3i} + \frac{1}{3-2i} = \frac{2-3i}{13} + \frac{3+2i}{13} = \frac{5-i}{13}$$
$$\frac{2}{z_1} = \frac{5-i}{13}$$

Let $z_1 = a + bi$
$$\frac{2}{a+bi} = \frac{5-i}{13}$$

$$26 = (5-i)(a+bi)$$
$$26 + (0)i = 5a + 5bi - ai + b$$
$$26 + (0)i = (5a+b) + (-a+5b)i$$

...Continued on the next page

or (a) Scale 15D (0, 4, 7, 11, 15)

Low Partial Credit:
- One complex number correct

Mid Partial Credit:
- Two complex numbers correct

High Partial Credit:
- Correct use of conjugate in 2/(a + bi) = (5−i)/13 13

$\Rightarrow 5a + b = 26$ ...(i) and $-a + 5b = 0$ ...(ii)

(i): $5a + b = 26$
(ii): $-5a + 25b = 0$

$26b = 26$

$b = 1$

From (ii): $5b = a$

$\Rightarrow a = 5$

$z_1 = 5 + i$

(b) Let ω be a complex number such that $\omega^n = 1$, $\omega \neq 1$, and $S = 1 + \omega + \omega^2 + \cdots + \omega^{n-1}$. Use the formula for the sum of a finite geometric series to write the value of S in its simplest form.

$S = 1 + \omega + \omega^2 + \cdots + \omega^{n-1}$

$a = 1, \quad r = \omega$

$$S = \frac{1(1-\omega^n)}{1-\omega} = \frac{1(1-1)}{1-\omega} = 0$$

(b) Scale 10C (0, 4, 8, 10)

Low Partial Credit:
- Correct Geometric Progression formula
- Correct first term
- Correct ratio

High Partial Credit:
- Values substituted in formula