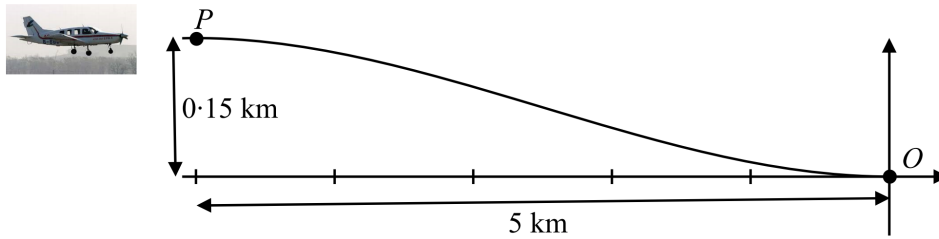


Question 7**(50 marks)**

A plane is flying horizontally at P at a height of 150 m above level ground when it begins its descent. P is 5 km, horizontally, from the point of touchdown O . The plane lands horizontally at O .



Taking O as the origin, $(x, f(x))$ approximately describes the path of the plane's descent where $f(x) = 0.0024x^3 + 0.018x^2 + cx + d$, $-5 \leq x \leq 0$, and both x and $f(x)$ are measured in km.

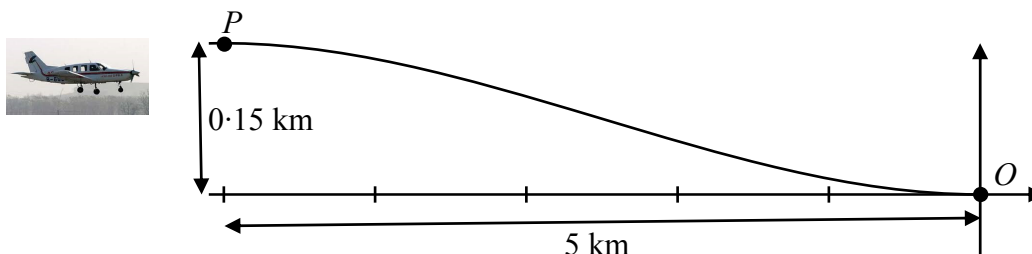
- (a) (i) Show that $d = 0$.
- (ii) Using the fact that P is the point $(-5, 0.15)$, or otherwise, show that $c = 0$.
- (b) (i) Find the value of $f'(x)$, the derivative of $f(x)$, when $x = -4$.
- (ii) Use your answer to part (b) (i) above to find the angle at which the plane is descending when it is 4 km from touchdown. Give your answer correct to the nearest degree.
- (c) Show that $(-2.5, 0.075)$ is the point of inflection of the curve $y = f(x)$.
- (d) (i) If (x, y) is a point on the curve $y = f(x)$, verify that $(-x-5, -y+0.15)$ is also a point on $y = f(x)$.
- (ii) Find the image of $(-x-5, -y+0.15)$ under symmetry in the point of inflection.

Answer **all three** questions from this section.

Question 7

(50 marks)

A plane is flying horizontally at P at a height of 150 m above level ground when it begins its descent. P is 5 km, horizontally, from the point of touchdown O . The plane lands horizontally at O .



Taking O as the origin, $(x, f(x))$ approximately describes the path of the plane's descent where $f(x) = 0.0024x^3 + 0.018x^2 + cx + d$, $-5 \leq x \leq 0$, and both x and $f(x)$ are measured in km.

- (a) (i) Show that $d = 0$.

$$f(x) = 0.0024x^3 + 0.018x^2 + cx + d$$

$$f(0) = 0 + 0 + 0 + d = 0 \Rightarrow d = 0$$

- (ii) Using the fact that P is the point $(-5, 0.15)$, or otherwise, show that $c = 0$.

$$f(x) = 0.0024x^3 + 0.018x^2 + cx$$

$$f(-5) = 0.0024(-5)^3 + 0.018(-5)^2 + c(-5) = 0.15$$

$$\Rightarrow 0.15 - 5c = 0.15 \Rightarrow c = 0$$

or

$$\text{The plane lands horizontally at } O \Rightarrow f'(x) = 0 \text{ when } x = 0$$

$$f'(x) = 0.0072x^2 + 0.036x + c$$

$$f'(0) = 0 + 0 + c = 0$$

$$\Rightarrow c = 0$$

- (b) (i) Find the value of $f'(x)$, the derivative of $f(x)$, when $x = -4$.

$$f(x) = 0.0024x^3 + 0.018x^2 + cx + d$$

$$f'(x) = 0.0072x^2 + 0.036x$$

$$f'(-4) = 0.0072(-4)^2 + 0.036(-4)$$

$$= -0.0288$$

- (ii) Use your answer to part (b) (i) above to find the angle at which the plane is descending when it is 4 km from touchdown. Give your answer correct to the nearest degree.

$$\tan \theta = f'(x) = -0.0288 \Rightarrow \theta = 178.3503^\circ$$

$$\text{Angle of descent } \alpha = 1.6497^\circ = 2^\circ$$

- (c) Show that $(-2.5, 0.075)$ is the point of inflection of the curve $y = f(x)$.

$$f'(x) = 0.0072x^2 + 0.036x$$

$$f''(x) = 0.0144x + 0.036 = 0$$

$$\Rightarrow x = -2.5$$

$$f(x) = 0.0024x^3 + 0.018x^2$$

$$f(-2.5) = 0.0024(-2.5)^3 + 0.018(-2.5)^2$$

$$= -0.0375 + 0.1125 = 0.075$$

$$(-2.5, 0.075)$$

- (d) (i) If (x, y) is a point on the curve $y = f(x)$, verify that $(-x-5, -y+0.15)$ is also a point on $y = f(x)$.

$$f(x) = 0.0024x^3 + 0.018x^2$$

$$f(-x-5) = 0.0024(-x-5)^3 + 0.018(-x-5)^2$$

$$= 0.0024(-x^3 - 15x^2 - 75x - 125) + 0.018(x^2 + 10x + 25)$$

$$= -0.0024x^3 - 0.018x^2 + 0x + 0.15$$

$$= -y + 0.15$$

- (ii) Find the image of $(-x-5, -y+0.15)$ under symmetry in the point of inflection.

Point: $(-x-5, -y+0.15)$
 Point of inflection: $(-2.5, 0.075)$

Change in x value: $(-2.5) - (-x-5) = x + 2.5$
 Change in y value: $0.075 - (-y+0.15) = y - 0.075$

Image of point of inflection:
 x value: $-2.5 + (x + 2.5) = x$
 y value: $0.075 + (y - 0.075) = y$
 $\Rightarrow (x, y)$ is image

or

Let (x, y) be the image.

$$\left(\frac{-x-5+x}{2}, \frac{-y+0.15+y}{2} \right) = (-2.5, 0.075), \text{ the point of inflection}$$