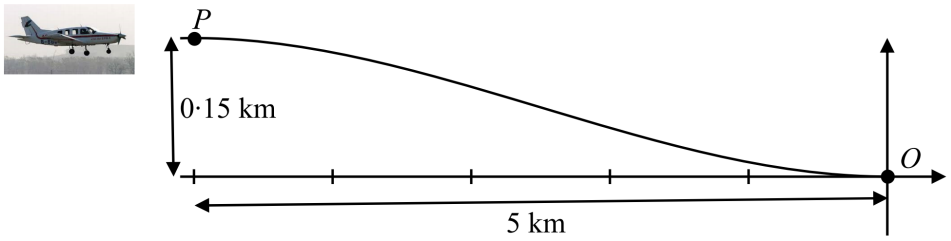
Question 7 **(50 marks)**

A plane is flying horizontally at P at a height of 150 m above level ground when it begins its descent. P is 5 km, horizontally, from the point of touchdown O. The plane lands horizontally at O.



Taking O as the origin, $(x,\ f(x))$ approximately describes the path of the plane's descent where $f(x) = 0{\cdot}0024x^3 + 0{\cdot}018x^2 + cx + d,\ -5 \le x \le 0,$ and both x and $f(x)$ are measured in km.

(a) **(i)** Show that $d = 0$.

(ii) Using the fact that P is the point $(-5,\ 0{\cdot}15),$ or otherwise, show that $c = 0$.

(b) **(i)** Find the value of $f'(x),$ the derivative of $f(x),$ when $x = -4$.

(ii) Use your answer to part **(b) (i)** above to find the angle at which the plane is descending when it is 4 km from touchdown. Give your answer correct to the nearest degree.

(c) Show that $(-2{\cdot}5,\ 0{\cdot}075)$ is the point of inflection of the curve $y = f(x)$.

(d) **(i)** If (x, y) is a point on the curve $y = f(x),$ verify that $(-x-5, -y+0{\cdot}15)$ is also a point on $y = f(x)$.

(ii) Find the image of $(-x-5, -y+0{\cdot}15)$ under symmetry in the point of inflection.

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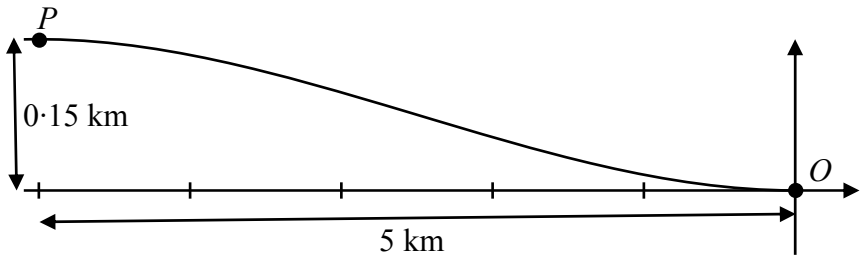
A plane is flying horizontally at P at a height of 150 m above level ground when it begins its descent. P is 5 km, horizontally, from the point of touchdown O. The plane lands horizontally at O.



Taking O as the origin, $(x,\ f(x))$ approximately describes the path of the plane's descent where $f(x)=0{\cdot}0024x^3+0{\cdot}018x^2+cx+d,\ -5\le x\le 0,$ and both x and $f(x)$ are measured in km.

(a) **(i)** Show that $d = 0$.

$$f(x) = 0\cdot0024x^3 + 0\cdot018x^2 + cx + d$$
$$f(0) = 0 + 0 + 0 + d = 0 \quad \Rightarrow \quad d = 0$$

(a)(i) Scale 5B (0, 2, 5)
Partial Credit:
• Recognises x = 0

(ii) Using the fact that P is the point $(-5,\ 0{\cdot}15),$ or otherwise, show that $c = 0$.

$$f(x) = 0\cdot0024x^3 + 0\cdot018x^2 + cx$$
$$f(-5) = 0\cdot0024(-5)^3 + 0\cdot018(-5)^2 + c(-5) = 0\cdot15$$
$$\Rightarrow\ 0\cdot15 - 5c = 0\cdot15 \quad \Rightarrow \quad c = 0$$

(a)(ii) Scale 5B (0, 2, 5) – 1st solution

Partial Credit: • Uses x=−5or f(x)=0.15

Full credit: • Begins with c = 0 and shows f(−5) = 0.15 or similar

or

The plane lands horizontally at $O \ \Rightarrow\ f'(x) = 0$ when $x = 0$

$$f'(x) = 0\cdot0072x^2 + 0\cdot036x + c$$
$$f'(0) = 0 + 0 + c = 0$$
$$\Rightarrow c = 0$$

(a)(ii) Scale 5B (0, 2, 5) – 2nd solution
Partial Credit:
• Uses x = −5
• Gets f'(x)
• Uses f '(x) = 0 when x = 0

(b) **(i)** Find the value of $f'(x),$ the derivative of $f(x),$ when $x = -4$.

$$f(x) = 0\cdot0024x^3 + 0\cdot018x^2 + cx + d$$
$$f'(x) = 0\cdot0072x^2 + 0\cdot036x$$
$$f'(-4) = 0\cdot0072\,(-4)^2 + 0\cdot036\,(-4)$$
$$= -0\cdot0288$$

(b)(i) Scale 10C (0, 3, 7, 10)

Low Partial Credit:
• Any term correctly differentiated.

High Partial Credit: • Correct differentiation

Full credit: • −18/625 is a correct answer

(ii) Use your answer to part **(b) (i)** above to find the angle at which the plane is descending when it is 4 km from touchdown. Give your answer correct to the nearest degree.

$\tan\theta = f'(x) = -0\cdot0288 \Rightarrow \theta = 178\cdot3503°$
Angle of descent $\alpha = 1\cdot6497° = 2°$

(b)(ii) Scale 5B (0, 2, 5) Partial Credit:
• Recognition of connection between slope and tan θ • Any right angled triangle

(c) Show that $(-2\cdot5,\ 0\cdot075)$ is the point of inflection of the curve $y = f(x)$.

$f'(x) = 0\cdot0072x^2 + 0\cdot036x$
$f''(x) = 0\cdot0144x + 0\cdot036 = 0$
$\Rightarrow x = -2\cdot5$
$f(x) = 0\cdot0024x^3 + 0\cdot018x^2$
$f(-2\cdot5) = 0\cdot0024(-2\cdot5)^3 + 0\cdot018(-2\cdot5)^2$
$= -0\cdot0375 + 0\cdot1125 = 0\cdot075$
$(-2\cdot5, 0\cdot075)$

(c) Scale 10D (0, 2, 5, 8, 10)

Low Partial Credit:
• Some correct differentiation of f '(x)
• Mention of f '(x)

Mid Partial Credit:
• Correct f ''(x) = 0

High Partial Credit:
• Value of x substituted

(d) **(i)** If (x, y) is a point on the curve $y = f(x)$, verify that $(-x-5, -y+0\cdot15)$ is also a point on $y = f(x)$.

$f(x) = 0\cdot0024x^3 + 0\cdot018x^2$
$f(-x-5) = 0\cdot0024(-x-5)^3 + 0\cdot018(-x-5)^2$
$= 0\cdot0024(-x^3 - 15x^2 - 75x - 125) + 0\cdot018(x^2 + 10x + 25)$
$= -0\cdot0024x^3 - 0\cdot018x^2 + 0x + 0\cdot15$
$= -y + 0\cdot15$

(d)(i) Scale 5C (0, 2, 4, 5)

Low Partial Credit:
• Some correct substitution

High Partial Credit:
• Correct expansions

(ii) Find the image of $(-x-5, -y+0\cdot15)$ under symmetry in the point of inflection.

Point: $(-x-5,\ -y+0\cdot15)$
Point of inflection: $(-2\cdot5,\ 0\cdot075)$

Change in x value: $(-2\cdot5)-(-x-5) = x+2\cdot5$
Change in y value: $0\cdot075-(-y+0\cdot15) = y-0\cdot075$

Image of point of inflection:
x value: $-2\cdot5+(x+2\cdot5) = x$
y value: $0\cdot075+(y-0\cdot075) = y$
$\Rightarrow (x, y)$ is image

(d)(ii) Scale 10C (0, 4, 8, 10) –
NOTE: two solutions 1st solution

Low Partial Credit:
• Work leading to change in x -value or y -value

High Partial Credit:
• Correct change in x and y values

or

Let (x, y) be the image.

$\left(\frac{-x-5+x}{2}, \frac{-y+0\cdot15+y}{2}\right) = (-2\cdot5, 0\cdot075)$, the point of inflection

(d)(ii) Scale 10C (0, 4, 8, 10) –
or 2nd solution

Low Partial Credit:
• Uses (x, y) as image, and no more

High Partial Credit:
• Effort at calculating mid-point

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