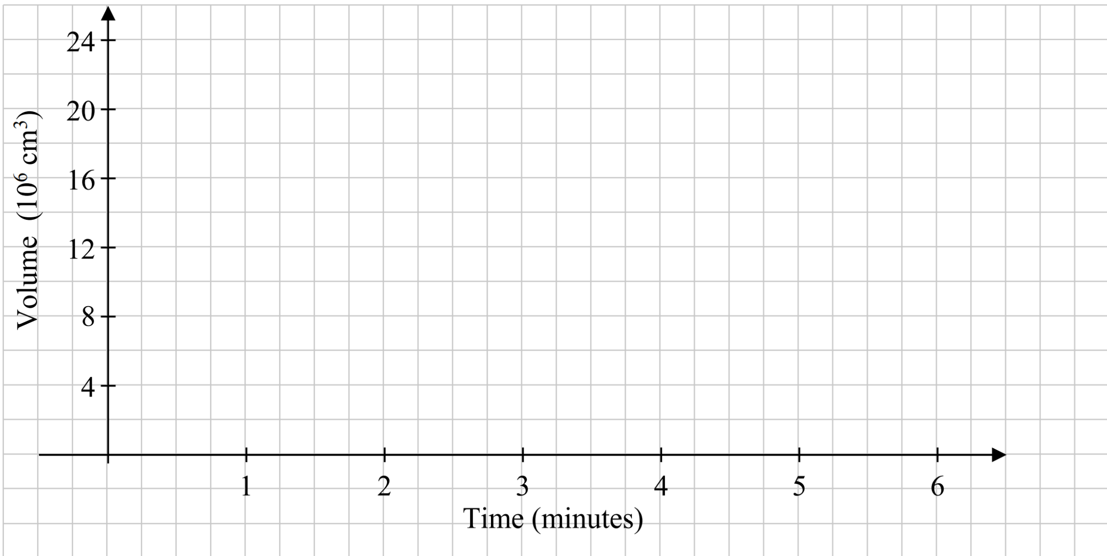
**Question 8** **(50 marks)**

An oil-spill occurs off-shore in an area of calm water with no currents. The oil is spilling at a rate of $4\times10^{6}$ cm$^{3}$ per minute. The oil floats on top of the water.

**(a)** **(i)** Complete the table below to show the total volume of oil on the water after each of the first 6 minutes of the oil-spill.

| Time (minutes) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Volume ($10^6$ cm$^3$) | | 8 | | | | |

**(ii)** Draw a graph to show the total volume of oil on the water over the first 6 minutes.




**(iii)** Write an equation for $V(t)$, the volume of oil on the water, in cm$^3$, after $t$ minutes.

**(b)** The spilled oil forms a circular oil slick **1 millimetre** thick.

**(i)** Write an equation for the volume of oil in the slick, in cm$^3$, when the radius is $r$ cm.

**(ii)** Find the rate, in cm per minute, at which the radius of the oil slick is increasing when the radius is 50 m.

**(c)** Show that the area of water covered by the oil slick is increasing at a constant rate of $4\times10^{7}$ cm$^2$ per minute.

**(d)** The nearest land is 1 km from the point at which the oil-spill began. Find how long it will take for the oil slick to reach land. Give your answer correct to the nearest hour.

**Question 8** **(50 marks)**

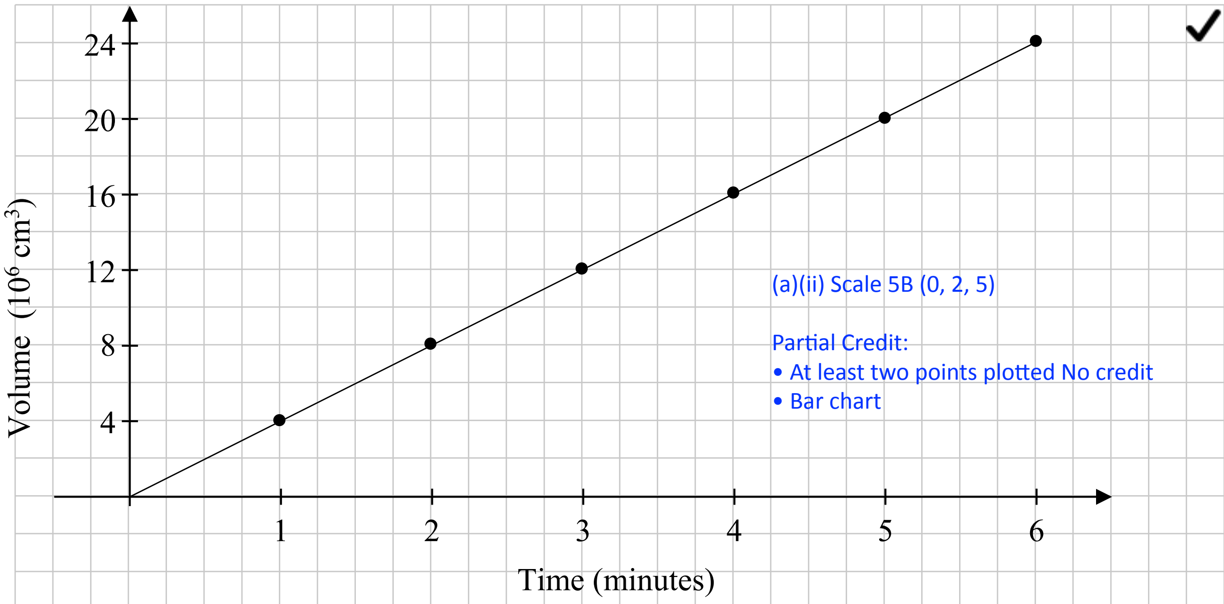
An oil-spill occurs off-shore in an area of calm water with no currents. The oil is spilling at a rate of $4\times10^6$ cm$^3$ per minute. The oil floats on top of the water.

**(a)** **(i)** Complete the table below to show the total volume of oil on the water after each of the first 6 minutes of the oil-spill.

(a)(i) Scale 5B (0, 2, 5) Partial Credit: • One correct box

| Time (minutes) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Volume ($10^6$ cm$^3$) | 4 | 8 | 12 | 16 | 20 | 24 |

**(ii)** Draw a graph to show the total volume of oil on the water over the first 6 minutes.




(a)(ii) Scale 5B (0, 2, 5)

Partial Credit:
- At least two points plotted No credit
- Bar chart

**(iii)** Write an equation for $V(t)$, the volume of oil on the water, in cm$^3$, after $t$ minutes.

Line, slope $4\times10^6$, passing through (0, 0).

$V(t)=\left(4\times10^6\right)t$

(a)(iii) Scale 5B (0, 2, 5)
Partial Credit:
- Incomplete equation for volume • any function of t

**(b)** The spilled oil forms a circular oil slick **1 millimetre** thick.

**(i)** Write an equation for the volume of oil in the slick, in cm$^3$, when the radius is $r$ cm.

$$
\begin{aligned}
V &= \pi r^2 h \\
&= \pi r^2 (0\cdot 1) \\
&= 0\cdot 1\pi r^2 \text{ cm}^3
\end{aligned}
$$

(b)(i) Scale 5B (0, 2, 5)

Partial Credit:
- Correct volume formula
- Converting mm to cm

**(ii)** Find the rate, in cm per minute, at which the radius of the oil slick is increasing when the radius is 50 m.

$$\frac{dV}{dt} = 4\times10^6 \text{ cm}^3 \text{ per minute}$$
$$V = \pi r^2 h \text{ where } h = 0\cdot1 \text{ cm}$$
$$\frac{dV}{dr} = 2\pi rh$$
$$\frac{dV}{dr} = 0\cdot2\pi r$$
$$\frac{dr}{dt} = \frac{dr}{dV}\frac{dV}{dt} = \frac{1}{0\cdot2\pi r}\times4\times10^6$$
$$= \frac{4\times10^6}{0\cdot2\pi(5000)} = 1273\cdot3 \text{ cm per minute}$$

(b)(ii) Scale 10D (0, 2, 5, 8, 10)

Low Partial Credit:
- Mentions a relevant rate of change.

Mid Partial Credit:
- Gets dr/dt from dV/dr and dV/dt
- Writing down chain rule.

High Partial Credit:
- Substitution of values

**(c)** Show that the area of water covered by the oil slick is increasing at a constant rate of $4\times10^7$ cm$^2$ per minute.

$$A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$$
$$\frac{dA}{dt} = \frac{dA}{dr}\frac{dr}{dt} = 2\pi r\frac{4\times10^6}{0\cdot2\pi r} = 4\times10^7 \text{ cm}^2 \text{ per minute}$$

(c) Scale 10C (0, 4, 8, 10) –
NOTE: two solutions 1st solution
Low Partial Credit:
- Mentions relevant rate of change.

High Partial Credit: states chain rule

**or**

$$(0\cdot1)\pi r^2 = (4\times10^6)t$$
$$\Rightarrow A = \pi r^2 = (4\times10^7)t$$
$$\frac{dA}{dt} = 4\times10^7$$

(c) Scale 10C (0, 4, 8, 10) – 2nd solution
Low Partial Credit:
- Effort to establish value of A

High Partial Credit:
- A in terms of t

Note: Must use calculus to get any credit.

**(d)** The nearest land is 1 km from the point at which the oil-spill began. Find how long it will take for the oil slick to reach land. Give your answer correct to the nearest hour.

$$A = \pi r^2 = \pi(10^5)^2 = \pi10^{10} \text{ cm}^2$$
$$t = \frac{\pi10^{10}}{4\times10^7} = \frac{\pi10^3}{4} = 785\cdot398 \text{ minutes}$$
$$= 13\cdot09 = 13 \text{ hours}$$

(d) Scale 10C (0, 4, 8, 10)

Low Partial Credit:
- r in centimetres
- Effort at expression of area

High Partial Credit:
- Correct expression for time