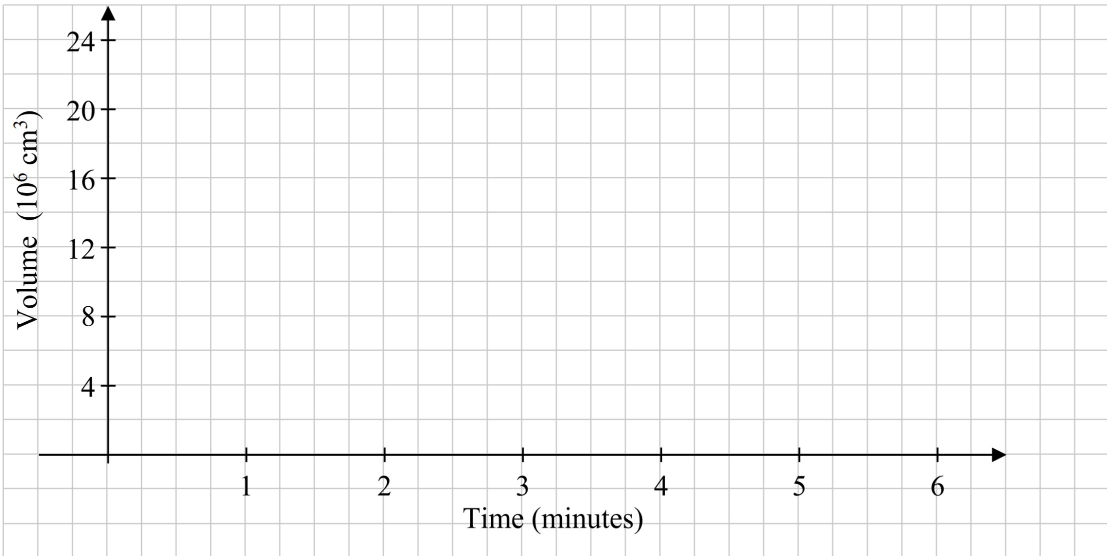
Question 8 **(50 marks)**

An oil-spill occurs off-shore in an area of calm water with no currents. The oil is spilling at a rate of 4×10^{6} cm ^{3} per minute. The oil floats on top of the water.

(a) **(i)** Complete the table below to show the total volume of oil on the water after each of the first 6 minutes of the oil-spill.

Time (minutes)	1	2	3	4	5	6
Volume (10^6 cm^3)		8				

(ii) Draw a graph to show the total volume of oil on the water over the first 6 minutes.



(iii) Write an equation for $V(t)$, the volume of oil on the water, in cm^3, after t minutes.

(b) The spilled oil forms a circular oil slick **1 millimetre** thick.

(i) Write an equation for the volume of oil in the slick, in cm^3, when the radius is r cm.

(ii) Find the rate, in cm per minute, at which the radius of the oil slick is increasing when the radius is 50 m.

(c) Show that the area of water covered by the oil slick is increasing at a constant rate of 4×10^{7} cm^2 per minute.

(d) The nearest land is 1 km from the point at which the oil-spill began. Find how long it will take for the oil slick to reach land. Give your answer correct to the nearest hour.

Question 8 **(50 marks)**

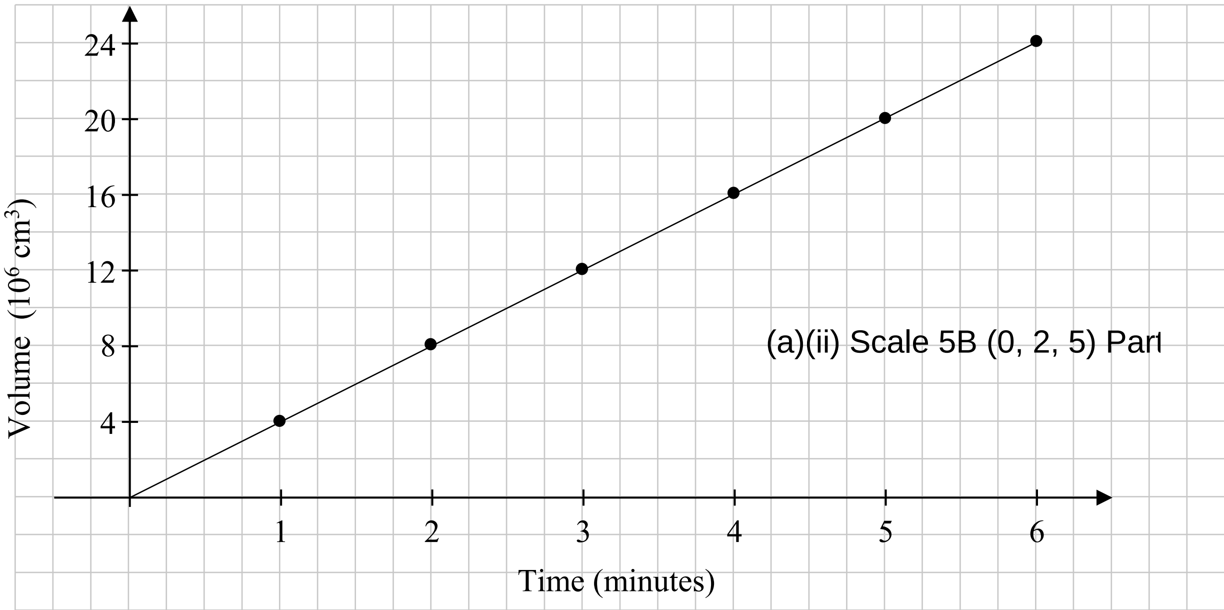
An oil-spill occurs off-shore in an area of calm water with no currents. The oil is spilling at a rate of 4×10^6 cm^3 per minute. The oil floats on top of the water.

(a) **(i)** Complete the table below to show the total volume of oil on the water after each of the first 6 minutes of the oil-spill.

(a)(i) Scale 5B (0, 2, 5) Partial Credit: • On

Time (minutes)	1	2	3	4	5	6
Volume (10^6 cm^3)	4	8	12	16	20	24

(ii) Draw a graph to show the total volume of oil on the water over the first 6 minutes.



(iii) Write an equation for $V(t)$, the volume of oil on the water, in cm^3, after t minutes.

Line, slope 4×10^6, passing through (0, 0).

$V(t) = \left(4 \times 10^6\right) t$

(b) The spilled oil forms a circular oil slick **1 millimetre** thick.

(i) Write an equation for the volume of oil in the slick, in cm^3, when the radius is r cm.

$$V = \pi r^2 h$$
$$= \pi r^2 (0 \cdot 1)$$
$$= 0 \cdot 1 \pi r^2 \text{ cm}^3$$

(ii) Find the rate, in cm per minute, at which the radius of the oil slick is increasing when the radius is 50 m.

$$\frac{dV}{dt} = 4\times10^{6} \text{ cm}^3 \text{ per minute}$$

$$V = \pi r^2 h \text{ where } h = 0\cdot1 \text{ cm}$$

$$\frac{dV}{dr} = 2\pi rh$$

$$\frac{dV}{dr} = 0\cdot 2\pi r$$

$$\frac{dr}{dt} = \frac{dr}{dV}\frac{dV}{dt} = \frac{1}{0\cdot2\pi r}\times4\times10^{6}$$

$$= \frac{4\times10^{6}}{0\cdot2\pi(5000)} = 1273\cdot3 \text{ cm per minute}$$

(c) Show that the area of water covered by the oil slick is increasing at a constant rate of 4×10^{7} cm^2 per minute.

$$A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$$

$$\frac{dA}{dt} = \frac{dA}{dr}\frac{dr}{dt} = 2\pi r\frac{4\times10^{6}}{0\cdot2\pi r} = 4\times10^{7} \text{ cm}^2 \text{ per minute}$$

or

$$(0\cdot1)\pi r^2 = (4\times10^{6})t$$

$$\Rightarrow A = \pi r^2 = (4\times10^{7})t$$

$$\frac{dA}{dt} = 4\times10^{7}$$

(d) The nearest land is 1 km from the point at which the oil-spill began. Find how long it will take for the oil slick to reach land. Give your answer correct to the nearest hour.

$$A = \pi r^2 = \pi(10^5)^2 = \pi10^{10} \text{ cm}^2$$

$$t = \frac{\pi10^{10}}{4\times10^{7}} = \frac{\pi10^{3}}{4} = 785\cdot398 \text{ minutes}$$

$$= 13\cdot09 = 13 \text{ hours}$$