Question 9 **(50 marks)**

The approximate length of the day in Galway, measured in hours from sunrise to sunset, may be calculated using the function

$$f(t) = 12\cdot25 + 4\cdot75\sin\left(\frac{2\pi}{365}t\right),$$

where t is the number of days after March 21st and $\left(\frac{2\pi}{365}t\right)$ is expressed in radians.

(a) Find the length of the day in Galway on June 5th (76 days after March 21st). Give your answer in hours and minutes, correct to the nearest minute.

(b) Find a date on which the length of the day in Galway is approximately 15 hours.

(c) Find $f'(t)$, the derivative of $f(t)$.

(d) Hence, or otherwise, find the length of the longest day in Galway.

(e) Use integration to find the average length of the day in Galway over the six months from March 21st to September 21st (184 days). Give your answer in hours and minutes, correct to the nearest minute.

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(a) Find the length of the day in Galway on June 5th (76 days after March 21st). Give your answer in hours and minutes, correct to the nearest minute.

$$f(t) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right)$$

$$f(76) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365} \times 76\right)$$

$= 12{\cdot}25 + 4{\cdot}587 = 16{\cdot}837 = 16$ hours 50 minutes


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(b) Find a date on which the length of the day in Galway is approximately 15 hours.

$$f(t) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right) = 15$$

$$\Rightarrow \sin\left(\frac{2\pi}{365}t\right) = 0 \cdot 578947$$

$$\Rightarrow \frac{2\pi}{365}t = 0 \cdot 6174371$$

$$\Rightarrow t = 35 \cdot 87$$

36 days after March 21 is April 26.


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(c) Find $f'(t)$, the derivative of $f(t)$.

$$f(t) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right)$$

$$f'(t) = 0 + 4 \cdot 75 \times \frac{2\pi}{365} \cos\left(\frac{2\pi}{365}t\right)$$

$$= \frac{9 \cdot 5\pi}{365} \cos\left(\frac{2\pi}{365}\right)t$$


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(d) Hence, or otherwise, find the length of the longest day in Galway.

$f(t)$ is a maximum when $\sin\left(\frac{2\pi}{365}t\right)$ is a maximum of 1.

$t = 12\cdot25 + 4\cdot75 = 17$ hours

or

$$f'(t) = 0 \Rightarrow \frac{9\cdot5\pi}{365}\cos\left(\frac{2\pi}{365}t\right) = 0$$
$$\Rightarrow \cos\left(\frac{2\pi}{365}t\right) = 0$$
$$\Rightarrow \frac{2\pi}{365}t = \frac{\pi}{2}$$
$$\Rightarrow t = \frac{365}{4} = 91\cdot25$$
$$f(91\cdot25) = 12\cdot25 + 4\cdot75\sin\left(\frac{2\pi}{365}\times 91\cdot25\right)$$
$$= 12\cdot25 + 4\cdot75\sin\frac{\pi}{2}$$
$$= 17 \text{ hours}$$

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(e) Use integration to find the average length of the day in Galway over the six months from March 21st to September 21st (184 days). Give your answer in hours and minutes, correct to the nearest minute.

$$\frac{1}{b-a}\int_a^b f(x)dx = \frac{1}{184}\int_0^{184}\left(12\cdot25 + 4\cdot75\sin\left(\frac{2\pi}{365}t\right)\right)dt$$
$$= \frac{1}{184}\left[12\cdot25t - 4\cdot75\times\frac{365}{2\pi}\cos\left(\frac{2\pi}{365}t\right)\right]_0^{184}$$
$$= \frac{1}{184}\left[(2254 + 275\cdot843) - (0 - 275\cdot934)\right]$$
$$= \frac{1}{184}[2805\cdot777]$$
$$= 15\cdot24879$$
$$= 15 \text{ hours } 15 \text{ minutes}$$