## (50 marks)

## **Question 9**

The approximate length of the day in Galway, measured in hours from sunrise to sunset, may be calculated using the function

$$f(t) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right),$$

where *t* is the number of days after March  $21^{\text{st}}$  and  $\left(\frac{2\pi}{365}t\right)$  is expressed in radians.

- (a) Find the length of the day in Galway on June 5<sup>th</sup> (76 days after March 21<sup>st</sup>). Give your answer in hours and minutes, correct to the nearest minute.
- (b) Find a date on which the length of the day in Galway is approximately 15 hours.
- (c) Find f'(t), the derivative of f(t).
- (d) Hence, or otherwise, find the length of the longest day in Galway.
- (e) Use integration to find the average length of the day in Galway over the six months from March 21<sup>st</sup> to September 21<sup>st</sup> (184 days). Give your answer in hours and minutes, correct to the nearest minute.

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(a) Find the length of the day in Galway on June 5<sup>th</sup> (76 days after March 21<sup>st</sup>). Give your answer in hours and minutes, correct to the nearest minute.

$$f(t) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right)$$

$$f(76) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}\times76\right)$$

$$= 12 \cdot 25 + 4 \cdot 587 = 16 \cdot 837 = 16 \text{ hours } 50 \text{ minutes}$$
(a) Scale 10C (0, 4, 8, 10)  
Low Partial Credit:  
• Uses t = 76  
High Partial Credit:  
• Correct substitution  
Note: Using pi=90 degrees one error, but  
do not penalise again in (b)

(b) Find a date on which the length of the day in Galway is approximately 15 hours.

$$f(t) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right) = 15$$

$$\Rightarrow \sin\left(\frac{2\pi}{365}t\right) = 0 \cdot 578947$$

$$\Rightarrow \frac{2\pi}{365}t = 0 \cdot 6174371$$

$$\Rightarrow t = 35 \cdot 87$$
36 days after March 21 is April 26.  
(b) Scale 10C (0, 4, 8, 10)  
Low Partial Credit:  
• Correct f (t)  
• substituted.  
High Partial Credit:  
• Correct equation with t only  
Note: Accept 35 or 36 substituted  
correctly and tested.

(c) Find f'(t), the derivative of f(t).

$$f(t) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right)$$

$$f'(t) = 0 + 4 \cdot 75 \times \frac{2\pi}{365} \cos\left(\frac{2\pi}{365}t\right)$$

$$= \frac{9 \cdot 5\pi}{365} \cos\left(\frac{2\pi}{365}t\right)$$
(c) Scale 10B (0, 5, 10)
Partial Credit:
• Any correct differentiation (note: '0' could be correct differentiation here)
Note: Substituting 180° for pi one error

$$f(t)$$
 is a maximum when  $sin\left(\frac{2\pi}{365}t\right)$  is a maximum of 1.  
 $t = 12 \cdot 25 + 4 \cdot 75 = 17$  hours

or

$$f'(t) = 0 \Rightarrow \frac{9 \cdot 5\pi}{365} \cos\left(\frac{2\pi}{365}t\right) = 0$$

$$\Rightarrow \cos\left(\frac{2\pi}{365}t\right) = 0$$

$$\Rightarrow \frac{2\pi}{365}t = \frac{\pi}{2}$$

$$\Rightarrow t = \frac{365}{4} = 91 \cdot 25$$

$$f(91 \cdot 25) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365} \times 91 \cdot 25\right)$$

$$= 12 \cdot 25 + 4 \cdot 75 \sin\frac{\pi}{2}$$

$$= 17 \text{ hours}$$
(d) Scale 10D (0, 2, 5, 8, 10) - both solutions  
Low Partial Credit:  
• f'(t)=0  
Mid Partial Credit:  
• Value of t  
High Partial Credit:  
• Value of t substituted into f (t)  
• f(t) maximum when sin \theta = 1  
Note: Accept 91 or 92 substituted and evaluated correctly for full marks.

(e) Use integration to find the average length of the day in Galway over the six months from March 21<sup>st</sup> to September 21<sup>st</sup> (184 days). Give your answer in hours and minutes, correct to the nearest minute.

$$\frac{1}{b-a} \int_{a}^{b} f(x)dx = \frac{1}{184} \int_{0}^{184} \left( 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right) \right) dt$$

$$= \frac{1}{184} \left[ 12 \cdot 25t - 4 \cdot 75 \times \frac{365}{2\pi} \cos\left(\frac{2\pi}{365}t\right) \right]_{0}^{184}$$

$$= \frac{1}{184} \left[ (2254 + 275 \cdot 843) - (0 - 275 \cdot 934) \right]$$

$$= \frac{1}{184} \left[ 2805 \cdot 777 \right]$$

$$= 15 \cdot 24879$$

$$= 15 \text{ hours 15 minutes}$$
(e) Scale 10D (0, 2, 5, 8, 10)
Low Partial Credit:
• Correct expression in x or t
• Correct limits
Mid Partial Credit:
• Correct integration
High Partial
Credit:
• Correct integration
High Partial
Credit:
• Correct integration
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