The approximate length of the day in Galway, measured in hours from sunrise to sunset, may be calculated using the function

$$
f(t)=12 \cdot 25+4 \cdot 75 \sin \left(\frac{2 \pi}{365} t\right)
$$

where $t$ is the number of days after March $21^{\text {st }}$ and $\left(\frac{2 \pi}{365} t\right)$ is expressed in radians.
(a) Find the length of the day in Galway on June $5^{\text {th }}$ ( 76 days after March $21^{\text {st }}$ ). Give your answer in hours and minutes, correct to the nearest minute.
(b) Find a date on which the length of the day in Galway is approximately 15 hours.
(c) Find $f^{\prime}(t)$, the derivative of $f(t)$.
(d) Hence, or otherwise, find the length of the longest day in Galway.
(e) Use integration to find the average length of the day in Galway over the six months from March $21^{\text {st }}$ to September $21^{\text {st }}$ (184 days). Give your answer in hours and minutes, correct to the nearest minute.

## Question 9

The approximate length of the day in Galway, measured in hours from sunrise to sunset, may be calculated using the function

$$
f(t)=12 \cdot 25+4 \cdot 75 \sin \left(\frac{2 \pi}{365} t\right),
$$

where $t$ is the number of days after March $21^{\text {st }}$ and $\left(\frac{2 \pi}{365} t\right)$ is expressed in radians.
(a) Find the length of the day in Galway on June $5^{\text {th }}$ (76 days after March $21^{\text {st }}$ ). Give your answer in hours and minutes, correct to the nearest minute.

$$
\begin{array}{ll}
f(t)=12 \cdot 25+4 \cdot 75 \sin \left(\frac{2 \pi}{365} t\right) & \text { (a) Scale } 10 C(0,4,8,10) \\
f(76)=12 \cdot 25+4 \cdot 75 \sin \left(\frac{2 \pi}{365} \times 76\right) & \begin{array}{l}
\text { Low Partial Credit: } \\
\bullet \text { Uses } t=76
\end{array} \\
=12 \cdot 25+4 \cdot 587=16 \cdot 837=16 \text { hours } 50 \text { minutes } & \begin{array}{l}
\text { High Partial Credit: } \\
\\
\text { • Correct substitution } \\
\text { Note: Using pi=90 degrees one error, but } \\
\text { do not penalise again in (b) }
\end{array} \\
\hline
\end{array}
$$

(b) Find a date on which the length of the day in Galway is approximately 15 hours.

$$
\begin{aligned}
& f(t)=12 \cdot 25+4 \cdot 75 \sin \left(\frac{2 \pi}{365} t\right)=15 \\
& \Rightarrow \sin \left(\frac{2 \pi}{365} t\right)=0 \cdot 578947 \\
& \Rightarrow \frac{2 \pi}{365} t=0 \cdot 6174371 \\
& \Rightarrow t=35.87
\end{aligned}
$$

(b) Scale $10 \mathrm{C}(0,4,8,10)$

Low Partial Credit:

- Correct f (t)
- substituted.

High Partial Credit:

- Correct equation with t only

36 days after March 21 is April 26.
Note: Accept 35 or 36 substituted correctly and tested.
(c) Find $f^{\prime}(t)$, the derivative of $f(t)$.

$$
\begin{aligned}
f(t) & =12 \cdot 25+4 \cdot 75 \sin \left(\frac{2 \pi}{365} t\right) \\
f^{\prime}(t) & =0+4 \cdot 75 \times \frac{2 \pi}{365} \cos \left(\frac{2 \pi}{365} t\right) \\
& =\frac{9 \cdot 5 \pi}{365} \cos \left(\frac{2 \pi}{365}\right) t
\end{aligned}
$$

(c) Scale 10B (0, 5, 10)

Partial Credit:

- Any correct differentiation (note: '0' could be correct differentiation here) Note: Substituting $180^{\circ}$ for pi one error
(d) Hence, or otherwise, find the length of the longest day in Galway.
$f(t)$ is a maximum when $\sin \left(\frac{2 \pi}{365} t\right)$ is a maximum of 1.
$t=12 \cdot 25+4 \cdot 75=17$ hours
or

$$
\begin{aligned}
& f^{\prime}(t)=0 \Rightarrow \frac{9 \cdot 5 \pi}{365} \cos \left(\frac{2 \pi}{365} t\right)=0 \\
& \Rightarrow \cos \left(\frac{2 \pi}{365} t\right)=0 \\
& \Rightarrow \frac{2 \pi}{365} t=\frac{\pi}{2} \\
& \Rightarrow t=\frac{365}{4}=91 \cdot 25 \\
& \begin{aligned}
f(91 \cdot 25) & =12 \cdot 25+4 \cdot 75 \sin \left(\frac{2 \pi}{365} \times 91 \cdot 25\right) \\
& =12 \cdot 25+4 \cdot 75 \sin \frac{\pi}{2} \\
& =17 \text { hours }
\end{aligned}
\end{aligned}
$$

(d) Scale 10D (0, 2, 5, 8, 10) - both solutions

Low Partial Credit:

- $f^{\prime}(t)=0$

Mid Partial Credit:

- Value of $t$

High Partial Credit:

- Value of $t$ substituted into $f(t)$
- $f(t)$ maximum when $\sin \theta=1$

Note: Accept 91 or 92 substituted and evaluated correctly for full marks.
(e) Use integration to find the average length of the day in Galway over the six months from March $21^{\text {st }}$ to September $21^{\text {st }}$ (184 days). Give your answer in hours and minutes, correct to the nearest minute.

$$
\begin{aligned}
& \frac{1}{b-a} \int_{a}^{b} f(x) d x=\frac{1}{184} \int_{0}^{184}\left(12 \cdot 25+4 \cdot 75 \sin \left(\frac{2 \pi}{365} t\right)\right) d t \\
& =\frac{1}{184}\left[12 \cdot 25 t-4 \cdot 75 \times \frac{365}{2 \pi} \cos \left(\frac{2 \pi}{365} t\right)\right]_{0}^{184} \\
& =\frac{1}{184}[(2254+275 \cdot 843)-(0-275 \cdot 934)] \\
& =\frac{1}{184}[2805 \cdot 777] \\
& =15 \cdot 24879 \\
& =15 \text { hours } 15 \text { minutes } \\
& \text { (e) Scale 10D (0, 2, 5, 8, 10) } \\
& \text { Low Partial Credit: } \\
& \text { - Correct expression in x or t } \\
& \text { - Correct formula } \\
& \text { - Correct limits } \\
& \text { Mid Partial Credit: } \\
& \text { - Any correct integration } \\
& \text { High Partial Credit: } \\
& \text { - Correct integration and } \\
& \text { effort at substitution } \\
& \text { Note: Integration with one } \\
& \text { error but finished correctly } \\
& \text { gets High Partial Credit. }
\end{aligned}
$$

