## Question 7

(50 marks)
A plane is flying horizontally at $P$ at a height of 150 m above level ground when it begins its descent. $P$ is 5 km , horizontally, from the point of touchdown $O$. The plane lands horizontally at $O$.


Taking $O$ as the origin, $(x, f(x))$ approximately describes the path of the plane's descent where $f(x)=0 \cdot 0024 x^{3}+0.018 x^{2}+c x+d,-5 \leq x \leq 0$, and both $x$ and $f(x)$ are measured in km.
(a) (i) Show that $d=0$.

(ii) Using the fact that $P$ is the point $(-5,0 \cdot 15)$, or otherwise, show that $c=0$.

(b) (i) Find the value of $f^{\prime}(x)$, the derivative of $f(x)$, when $x=-4$.

(ii) Use your answer to part (b) (i) above to find the angle at which the plane is descending when it is 4 km from touchdown. Give your answer correct to the nearest degree.

(c) Show that $(-2 \cdot 5,0 \cdot 075)$ is the point of inflection of the curve $y=f(x)$.

(d) (i) If $(x, y)$ is a point on the curve $y=f(x)$, verify that $(-x-5,-y+0 \cdot 15)$ is also a point on $y=f(x)$.

(ii) Find the image of $(-x-5,-y+0 \cdot 15)$ under symmetry in the point of inflection.


