Question 3 **(25 marks)**

(a) The co-ordinates of two points are $A(4, -1)$ and $B(7, t)$.

The line $l_1 : 3x - 4y - 12 = 0$ is perpendicular to AB. Find the value of t.

(b) Find, in terms of k, the distance between the point $P(10, k)$ and l_1.

(c) $P(10, k)$ is on a bisector of the angles between the lines l_1 and $l_2 : 5x + 12y - 20 = 0$.

(i) Find the possible values of k.

(ii) If $k > 0$, find the distance from P to l_1.

Question 3 **(25 marks)**

(a) The co-ordinates of two points are $A(4, -1)$ and $B(7, t)$.

The line $l_1 : 3x - 4y - 12 = 0$ is perpendicular to AB. Find the value of t.

$$\text{Slope } AB = \frac{t+1}{7-4} = \frac{t+1}{3} \qquad\qquad \text{Slope } l_1 = \frac{3}{4}$$

$$AB \perp l_1 \Rightarrow \frac{t+1}{3} \times \frac{3}{4} = -1 \Rightarrow t+1 = -4 \Rightarrow t = -5$$

or

$$AB : 4x + 3y + c = 0$$
$$(4,-1) \in 4x + 3y + c = 0 \Rightarrow 16 - 3 + c = 0 \Rightarrow c = -13$$
$$\therefore 4(7) + 3(t) - 13 = 0 \Rightarrow t = -5$$

(b) Find, in terms of k, the distance between the point $P(10, k)$ and l_1.

$$d = \left| \frac{3(10) - 4k - 12}{\sqrt{3^2 + 4^2}} \right| = \left| \frac{18 - 4k}{5} \right|$$

(c) $P(10, k)$ is on a bisector of the angles between the lines l_1 and $l_2 : 5x + 12y - 20 = 0$.

(i) Find the possible values of k.

$$\left| \frac{18 - 4k}{5} \right| = \left| \frac{50 + 12k - 20}{\sqrt{5^2 + 12^2}} \right|$$
$$\Rightarrow \left| \frac{18 - 4k}{5} \right| = \left| \frac{30 + 12k}{13} \right|$$
$$\Rightarrow 13(18 - 4k) = \pm 5(30 + 12k)$$
$$\Rightarrow -112k = -84 \qquad \text{or} \qquad 8k = -384$$
$$\Rightarrow k = \tfrac{3}{4} \qquad \text{or} \qquad k = -48$$

(ii) If $k > 0$, find the distance from P to l_1.

$$k = \frac{3}{4} \Rightarrow d = \left| \frac{18 - 4\left(\frac{3}{4}\right)}{5} \right| = 3$$

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Question 3 **(25 marks)**

(a) **Scale 10D (0, 2, 5, 8,10)**
Low Partial Credit:
- Slope AB or l_1

Mid Partial Credit:
- Both slopes found

High Partial Credit:
- Slopes linked to perpendicularity

(b) **Scale 10C (0, 4, 8, 10)**
Low *Partial Credit:*
- Relevant formula with some correct substitution

High Partial Credit
- Substitution into formula fully correct

(c) **Scale 5D (0, 2, 3, 4, 5)**
Low *Partial Credit:*
- Relevant formula with some correct substitution

Mid Partial Credit:
- One value for k found
- Work indicating two values for k

High Partial Credit:
- Both values of k
- Positive value for k evaluated and distance calculated