Question 5 **(25 marks)**

(a) Prove that $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$.

(b) Find all the values of x for which $\sin(3x) = \dfrac{\sqrt{3}}{2}$, $0 \le x \le 360$, x in degrees.

Question 5 **(25 marks)**

(a) Prove that $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$.

$$\tan(A+B)=\frac{\sin(A+B)}{\cos(A+B)}$$
$$=\frac{\sin A\cos B+\cos A\sin B}{\cos A\cos B-\sin A\sin B}$$
$$=\frac{\dfrac{\sin A\cos B}{\cos A\cos B}+\dfrac{\cos A\sin B}{\cos A\cos B}}{\dfrac{\cos A\cos B}{\cos A\cos B}-\dfrac{\sin A\sin B}{\cos A\cos B}}$$
$$=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

or

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{\dfrac{\sin A}{\cos A}+\dfrac{\sin B}{\cos B}}{1-\dfrac{\sin A\sin B}{\cos A\cos B}}=$$
$$\frac{\dfrac{\sin A\cos B+\cos A\sin B}{\cos A\cos B}}{\dfrac{\cos A\cos B-\sin A\sin B}{\cos A\cos B}}=\frac{\sin A\cos B+\cos A\sin B}{\cos A\cos B-\sin A\sin B}=$$
$$\frac{\sin(A+B)}{\cos(A+B)}=\tan(A+B)$$

(b) Find all the values of x for which $\sin(3x)=\dfrac{\sqrt{3}}{2}$, $0\le x\le 360$, x in degrees.

$$\sin 3x=\frac{\sqrt{3}}{2}$$
$$\Rightarrow 3x=60°,\quad 120°,\quad 420°,\quad 480°,\quad 780°,\quad 840°$$
$$\Rightarrow x=20°,\quad 40°,\quad 140°,\quad 160°,\quad 260°,\quad 280°$$

or

$3x=60°+n(360°),\ n\in\mathbb{Z}$ or $3x=120°+n(360°),\ n\in\mathbb{Z}$
$x=20°+n(120°),\ n\in\mathbb{Z}$ or $x=40°+n(120°),\ n\in\mathbb{Z}$

$n=0\Rightarrow x=20°$ or $x=40°$
$n=1\Rightarrow x=140°$ or $x=160°$
$n=2\Rightarrow x=260°$ or $x=280°$

Question 5 **(25 marks)**

(a) Scale 15D (0, 4, 7, 11, 15)

Low *Partial Credit:*

- Tan function in terms of Sine and Cosine

Mid Partial Credit:

- sin (A+B) or cos (A+B) expanded
- Numerator or denominator in fraction form (method 2)

High Partial Credit:

- Numerator and denominator divided by cosA.cosB (Method 1)
- Both numerator and denominator expressed in form of a single fraction (Method 2)

(b) Scale 10D (0, 2, 5, 8, 10)

Low *Partial Credit:*

- One value for $3x$

Mid Partial Credit:

- One value for x
- Two or more values for $3x$

High Partial Credit:

- Three or more values for x