Question 7   **(40 marks)**
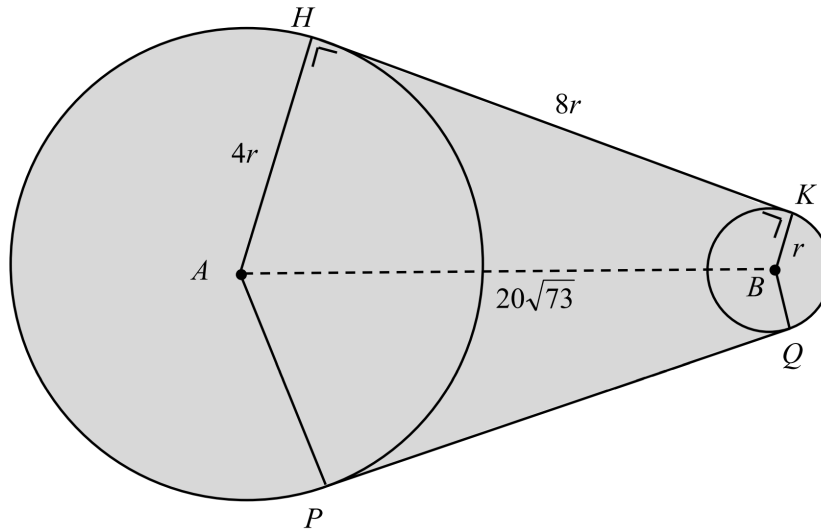
A flat machine part consists of two circular ends attached to a plate, as shown (diagram not to scale).
The sides of the plate, HK and PQ, are tangential to each circle.
The larger circle has centre A and radius $4r$ cm.
The smaller circle has centre B and radius r cm.
The length of $[HK]$ is $8r$ cm and $|AB| = 20\sqrt{73}$ cm.

(a) Find r, the radius of the smaller circle. (Hint: Draw $BT \parallel KH$, $T \in AH$.)

(b) Find the area of the quadrilateral $ABKH$.

(c) **(i)** Find $|\angle HAP|$, in degrees, correct to one decimal place.

      (ii) Find the area of the machine part, correct to the nearest cm^2.

Answer **all three** questions from this section.

Question 7   (40 marks)
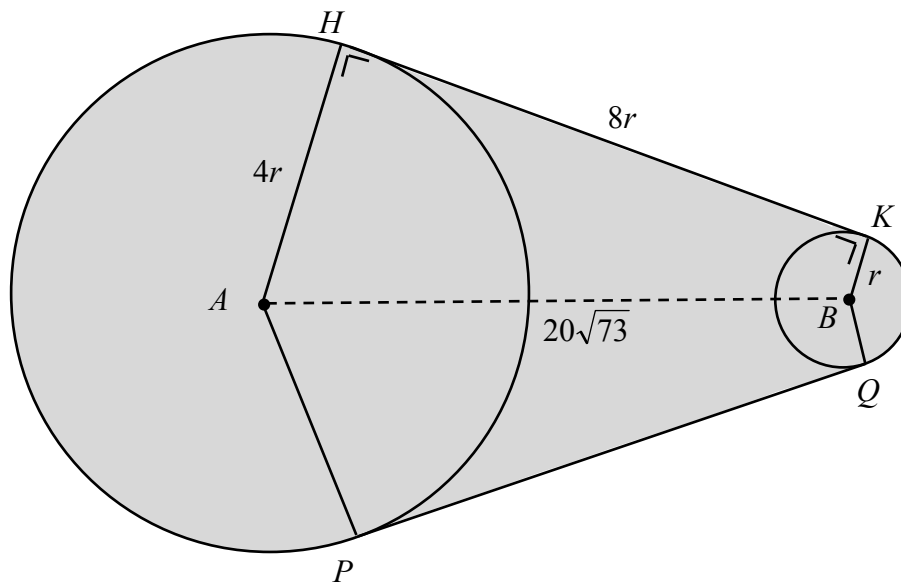
A flat machine part consists of two circular ends attached to a plate, as shown (diagram not to scale).
The sides of the plate, *HK* and *PQ*, are tangential to each circle.
The larger circle has centre *A* and radius 4*r* cm.
The smaller circle has centre *B* and radius *r* cm.
The length of [*HK*] is 8*r* cm and $|AB| = 20\sqrt{73}$ cm.

(a)   Find *r*, the radius of the smaller circle. (Hint: Draw $BT \parallel KH$, $T \in AH$.)

$$|AT|^2 + |BT|^2 = |AB|^2 \quad \Rightarrow \quad (3r)^2 + (8r)^2 = \left(20\sqrt{73}\right)^2$$
$$\Rightarrow \quad 9r^2 + 64r^2 = 29200$$
$$\Rightarrow \quad r^2 = 400 \quad \Rightarrow \quad r = 20 \text{ cm}$$

(b) Find the area of the quadrilateral *ABKH*.

$$| ABKH | = | BKHT | + | \Delta ABT |$$
$$= 20 \times 160 + \tfrac{1}{2}(60)(160)$$
$$= 8000 \ \text{cm}^2$$

(c) **(i)** Find $| \angle HAP |$, in degrees, correct to one decimal place.

$$\tan | \angle HAB | = \frac{160}{60} \quad \Rightarrow \quad | \angle HAB | = 69 \cdot 44°$$
$$\Rightarrow \quad | \angle HAP | = 138 \cdot 9°$$

(ii) Find the area of the machine part, correct to the nearest cm^2.

Area large sector *HAP* + 2 area *HABK* + area sector *KBQ*
$$= \pi (80)^2 \left(\frac{221 \cdot 1}{360} \right) + 2 \times 8000 + \pi (20)^2 \left(\frac{138 \cdot 9}{360} \right)$$
$$= 12348 \cdot 55 + 16000 + 484 \cdot 85$$
$$= 28833 \cdot 4$$
$$= 28833$$

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Section B

Question 7 (40 marks)

(a) **Scale 15C (0, 5, 12, 15)**
Low *Partial Credit:*
- *BT* drawn correctly
- Pythagoras formula with some correct substitution
- Recognising $|\angle ATB| = 90°$

High Partial Credit:
- Pythagoras formula fully substituted

(b) **Scale 15C (0, 5, 12, 15)**
Low *Partial Credit:*
- Indicates two areas
- Effort at area of rectangle only
- Effort at area of triangle only

High Partial Credit:
- Area of triangle correct
- Area of rectangle correct

(c)(i) Scale 5C (0, 2, 4, 5)
Low *Partial Credit:*
- $\tan \angle HAB = \dfrac{160}{60}$ or equivalent in sin or cos

High Partial Credit:
- $|\angle HAB|$ in degrees.

(c)(ii) Scale 5D (0, 2, 3, 4, 5)
Low *Partial Credit:*
- Effort at area of one region

Mid Partial Credit:
- Area of one sector with correct substitution

High Partial Credit:
- Area of two sectors with substitution correct in both.