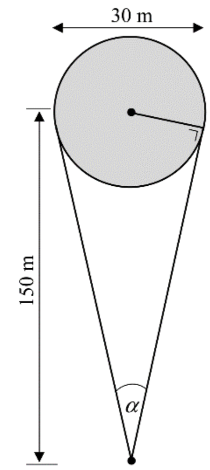
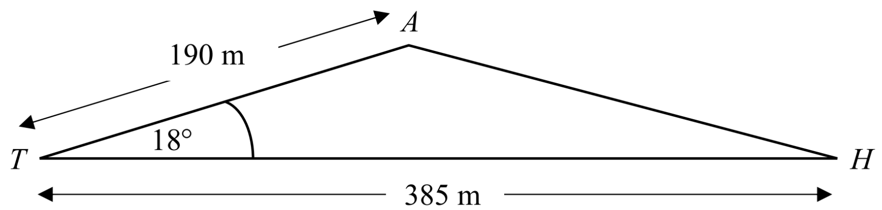
Question 9 **(45 marks)**

(a) Joan is playing golf. She is 150 m from the centre of a circular green of diameter 30 m. The diagram shows the range of directions in which Joan can hit the ball so that it could land on the green.
Find α , the measure of the angle of this range of directions.
Give your answer, in degrees, correct to one decimal place.



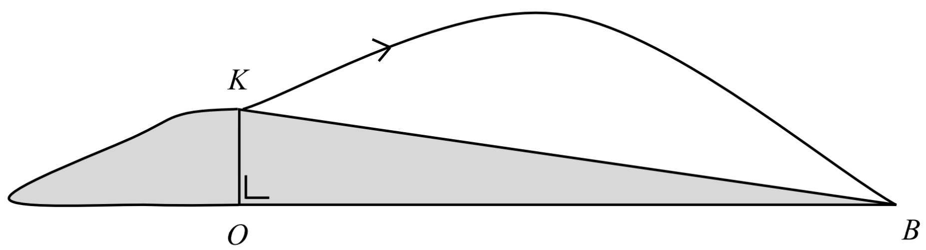
(b) At the next hole, Joan, at T, attempts to hit the ball in the direction of the hole H. Her shot is off target and the ball lands at A, a distance of 190 metres from T, where $|\angle ATH| = 18°$. $|TH|$ is 385 metres. Find $|AH|$, the distance from the ball to the hole, correct to the nearest metre.



(c) At another hole, where the ground is not level, Joan hits the ball from K, as shown. The ball lands at B. The height of the ball, in metres, above the horizontal line OB is given by

$$h = -6t^2 + 22t + 8$$

where t is the time in seconds after the ball is struck and h is the height of the ball.



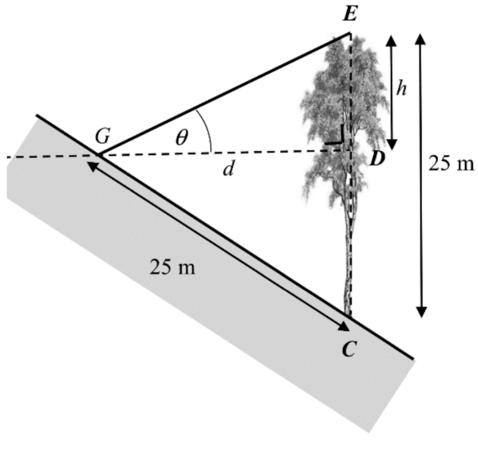
(i) Find the height of K above OB.

(ii) The horizontal speed of the ball over the straight distance $[OB]$ is a constant 38 m s^{-1}. Find the angle of elevation of K from B, correct to the nearest degree.

(d) At a later hole, Joan's first shot lands at the point G, on ground that is sloping downwards, as shown. A vertical tree, $[CE]$, 25 metres high, stands between G and the hole. The distance, $|GC|$, from the ball to the bottom of the tree is also 25 metres.

The angle of elevation at G to the top of the tree, E, is θ, where $\theta = \tan^{-1}(1/2)$.

The height of the top of the tree above the horizontal, GD, is h metres and $|GD| = d$ metres.

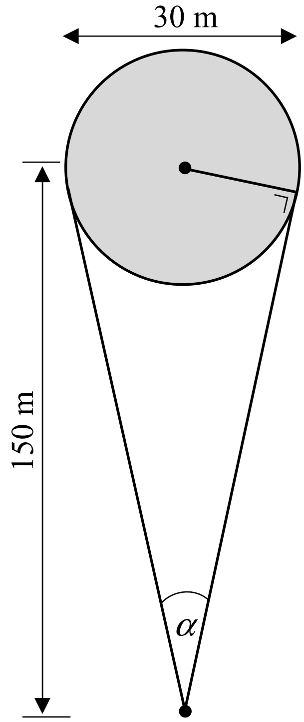


(i) Write d and $|CD|$ in terms of h.

(ii) Hence, or otherwise, find h.

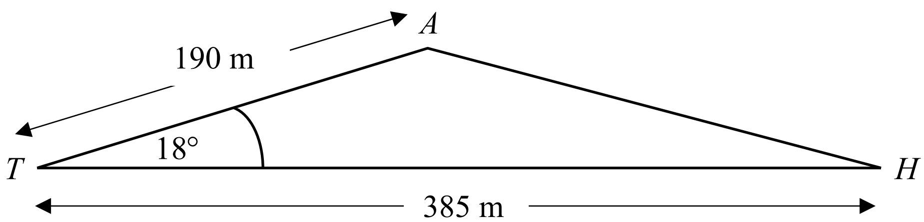
Question 9 **(45 marks)**

(a) Joan is playing golf. She is 150 m from the centre of a circular green of diameter 30 m. The diagram shows the range of directions in which Joan can hit the ball so that it could land on the green. Find α, the measure of the angle of this range of directions. Give your answer, in degrees, correct to one decimal place.



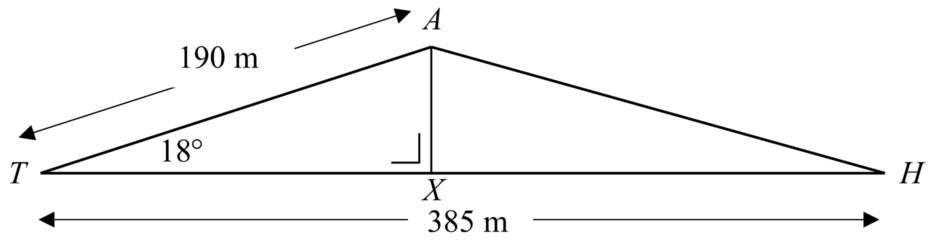
$$\sin \tfrac{1}{2}\alpha = \frac{15}{150} = 0\cdot 1$$
$$\Rightarrow \tfrac{1}{2}\alpha = 5\cdot 739^\circ$$
$$\Rightarrow \alpha = 11\cdot 478^\circ$$
$$\alpha = 11\cdot 5^\circ$$

(b) At the next hole, Joan, at T, attempts to hit the ball in the direction of the hole H. Her shot is off target and the ball lands at A, a distance of 190 metres from T, where $|\angle ATH| = 18^\circ$. $|TH|$ is 385 metres. Find $|AH|$, the distance from the ball to the hole, correct to the nearest metre.



$$|AH|^2 = 190^2 + 385^2 - 2(190)(385)\cos 18^\circ$$
$$= 36100 + 148225 - 139139\cdot 5683$$
$$= 45185\cdot 4317$$
$$|AH| = 212\cdot 57 = 213$$

or



Draw AX perpendicular to TH

triangle ATX: $\sin 18° = \dfrac{|AX|}{190} \Rightarrow |AX| = 58 \cdot 71$

$$\cos 18° = \frac{|TX|}{190} \Rightarrow |TX| = 180 \cdot 7$$

$$\Rightarrow |XH| = 204 \cdot 3$$

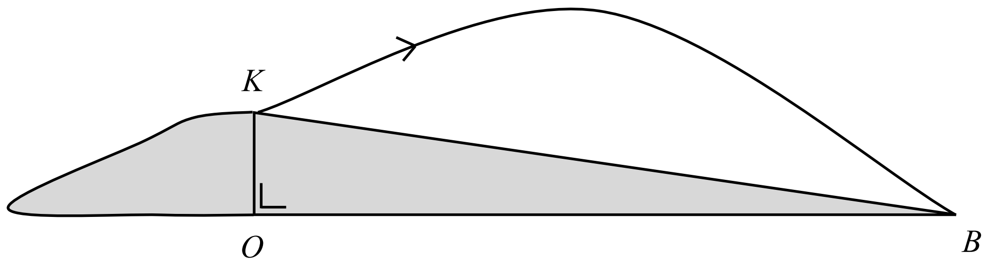
$$\Rightarrow |AH|^2 = (58 \cdot 71)^2 + (204 \cdot 3)^2$$

$$\Rightarrow |AH| = 212 \cdot 566 = 213$$

(c) At another hole, where the ground is not level, Joan hits the ball from K, as shown. The ball lands at B. The height of the ball, in metres, above the horizontal line OB is given by

$$h = -6t^2 + 22t + 8$$

where t is the time in seconds after the ball is struck and h is the height of the ball.



(i) Find the height of K above OB.

$h = -6t^2 + 22t + 8$
$t = 0 \Rightarrow h = 8$ m

(ii) The horizontal speed of the ball over the straight distance [OB] is a constant 38 m s^{-1}. Find the angle of elevation of K from B, correct to the nearest degree.

$$h = 0 \Rightarrow -6t^2 + 22t + 8 = 0$$
$$\Rightarrow (t-4)(-6t-2) = 0$$
$$\Rightarrow t = 4, \quad t = -\tfrac{1}{3}$$

$t = 4 \Rightarrow |OB| = 38 \times 4 = 152$ m

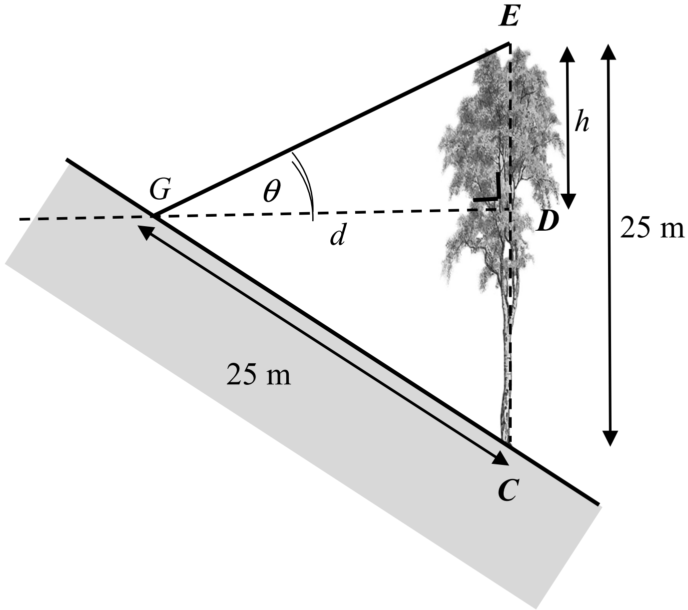
$$\tan|\angle OBK| = \frac{8}{152} = \frac{1}{19} \quad \Rightarrow \quad |\angle OBK| = 3\cdot01^\circ = 3^\circ$$

(d) At a later hole, Joan's first shot lands at the point G, on ground that is sloping downwards, as shown. A vertical tree, $[CE]$, 25 metres high, stands between G and the hole. The distance, $|GC|$, from the ball to the bottom of the tree is also 25 metres.

The angle of elevation at G to the top of the tree, E, is θ, where $\theta = \tan^{-1}\frac{1}{2}$.

The height of the top of the tree above the horizontal, GD, is h metres and $|GD| = d$ metres.

(i) Write d and $|CD|$ in terms of h.



$$\tan\theta = \frac{h}{d} = \frac{1}{2}$$
$$\Rightarrow d = 2h$$

$$|CD| = 25 - h$$

(ii) Hence, or otherwise, find h.

$$d^2 + |CD|^2 = 25^2$$
$$(2h)^2 + (25-h)^2 = 25^2$$
$$4h^2 + 625 - 50h + h^2 = 625$$
$$5h^2 - 50h = 0$$
$$h = 0, \quad h = 10$$
$$h = 10 \text{ m}$$

or

$$\theta = \tan^{-1}\tfrac{1}{2} = 26\cdot565^\circ$$
$$\Rightarrow \quad |GED| = 63\cdot435^\circ$$
$$\Rightarrow \quad |CGE| = 63\cdot435^\circ$$
$$\Rightarrow \quad |CGD| = 63\cdot435^\circ - 26\cdot565^\circ = 36\cdot87^\circ$$
$$\sin 36\cdot87 = \frac{25-h}{25} = 0\cdot6$$
$$\Rightarrow 25 - h = 15$$
$$\Rightarrow h = 10 \text{ m}$$

or

$$|\angle GCE = 53\cdot14^\circ| \Rightarrow \sin 53\cdot14^\circ = \frac{2h}{25}$$
$$\Rightarrow 0\cdot8 = \frac{2h}{25} \Rightarrow h = 10 \text{ m}$$

Question 9 **(45 marks)**

(a) Scale 10C (0, 4, 8, 10)

Low *Partial Credit:*

- Effort at expressing sine function in terms of 15 and 150
- Finds third side of triangle and makes effort to find an angle

High Partial Credit:

- Half angle found

(b) Scale 10C (0, 4, 8, 10)

Low *Partial Credit:*

- Cosine Rule with some correct substitution
- Effort at calculating $|AX|$ or $|TX|$

High Partial Credit:

- Cosine Rule substituted correctly
- Finds $|AX|$ and formulates for $|TX|$ (or vice versa)

(c)(i) Scale 5B (0, 2, 5)

Partial Credit:

- $t = 0$ indicated

Accept h = 8 m without work

(c)(ii) Scale 10C (0, 4, 8, 10)

Low *Partial Credit:*

- $h = 0$ indicated

High Partial Credit:

- $|OB|$ found for positive value for t

(d)(i) Scale 5B (0, 2, 5)

Partial Credit:

- $\dfrac{h}{d} = \dfrac{1}{2}$
- $|CD| = 25 - h$

(d)(ii) Scale 5D (0, 2, 3, 4, 5)

Low *Partial Credit:*

- Pythagoras with some correct substitution.
- Effort at evaluating θ

Mid Partial Credit:

- Pythagoras correctly substituted
- $\tan^{-1}\frac{1}{2}$ evaluated

High Partial Credit:

- Quadratic equation expanded correctly
- $\sin \angle CGD = \frac{|CD|}{|GC|}$ with $|\angle CGD|$ calculated
- $\sin \angle GCE = \frac{|GD|}{|GC|}$ with $|\angle GCE|$ calculated