Question 8 **(65 marks)**

In basketball, players often have to take free throws. When Michael takes his first free throw in any game, the probability that he is successful is 0·7.
For all subsequent free throws in the game, the probability that he is successful is:

- 0·8 if he has been successful on the previous throw
- 0·6 if he has been unsuccessful on the previous throw.

(a) Find the probability that Michael is successful (S) with all three of his first three free throws in a game.

P(S, S, S) =

(b) Find the probability that Michael is unsuccessful (U) with his first two free throws and successful with the third.

P(U, U, S) =

(c) List all the ways that Michael could be successful with his third free throw in a game and hence find the probability that Michael is successful with his third free throw.

(d) **(i)** Let p_n be the probability that Michael is successful with his n^{th} free throw in the game (and hence $(1-p_n)$ is the probability that Michael is unsuccessful with his n^{th} free throw).

Show that $p_{n+1} = 0{\cdot}6 + 0{\cdot}2\,p_n$.

(ii) Assume that p is Michael's success rate in the long run; that is, for large values of n, we have $p_{n+1} \approx p_n \approx p$.

Using the result from part **(d) (i)** above, or otherwise, show that $p = 0{\cdot}75$.

(e) For all positive integers n, let $a_n = p - p_n$, where $p = 0{\cdot}75$ as above.

(i) Use the ratio $\dfrac{a_{n+1}}{a_n}$ to show that a_n is a geometric sequence with common ratio $\dfrac{1}{5}$.

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(ii) Find the smallest value of n for which $p - p_n < 0{\cdot}00001$.

(f) You arrive at a game in which Michael is playing. You know that he has already taken many free throws, but you do not know what pattern of success he has had.

(i) Based on this knowledge, what is your estimate of the probability that Michael will be successful with his next free throw in the game?

Answer: _______________

(ii) Why would it **not** be appropriate to consider Michael's subsequent free throws in the game as a sequence of Bernoulli trials?