Question 1 **(25 marks)**

(a) $(-4+3i)$ is one root of the equation $az^2+bz+c=0$, where $a, b, c \in \mathbb{R}$, and $i^2=-1$.
Write the other root.

(b) Use De Moivre's Theorem to express $(1+i)^8$ in its simplest form.

(c) $(1+i)$ is a root of the equation $z^2+(-2+i)z+3-i=0$.
Find its other root in the form $m+ni$, where $m, n \in \mathbb{R}$, and $i^2=-1$.

Q1	Model Solution – 25 Marks	Marking Notes
(a)	$-4-3i$	Scale 5B (0, 2, 5) *Partial Credit:* • real or imaginary part correct
(b)	$r=\sqrt{1^2+1^2}=\sqrt{2}\qquad \theta=\frac{\pi}{4}$ $(1+i)^8=\left\{\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right\}^8$ $(1+i)^8=\{16(\cos 2\pi+i\sin 2\pi)\}$ $(1+i)^8$= 16(1) = 16	Scale 10C (0, 3, 7, 10) *Low Partial Credit:* • correct answer without use of De Moivre's • modulus **or** argument correct • formula • statement of De Moivre's *High Partial Credit:* • $16(\cos 2\pi+i\sin 2\pi)$ **Note:** *not De Moivre and incorrect answer merits 0 marks*
(c)	$z=\frac{(2-i)\pm\sqrt{(-2+i)^2-4(3-i)}}{2}$ $=\frac{(2-i)\pm\sqrt{4-4i-1-12+4i}}{2}$ $=\frac{2-i\pm\sqrt{-9}}{2}$ $=\frac{2-i\pm 3i}{2}$ $=1-2i$ **or** $1+i$ **Or** $ax^2+bx+c=0$ $x^2-\left(\frac{-b}{a}\right)x+\frac{c}{a}=0$ Sum of roots $=-\frac{b}{a}$ $1+i+z_1=2-i$ $z_1=1-2i$	Scale 10C (0, 3, 7, 10) *Low Partial Credit:* • root formula with some substitution *High Partial Credit* • formula fully substituted **Or** Scale 10C (0, 3, 7, 10) *Low Partial Credit:* • equation rearranged • $-\frac{b}{a}$ *High Partial Credit* • correct substitution

	Or $(z-1-i)(z-z_1)$ $= z^2 - z - zi - z.z_1 + z_1 + z_1 i$ $= z^2 - (1+i+z_1)z + z_1(1+i)$ $= z^2 + (-2+i)z + (3-i)$ $\Rightarrow z_1(1+i) = 3-i$ $z_1 = \frac{3-i}{1+i}.\frac{1-i}{1-i} = 1-2i$	**Or** Scale 10C (0, 3, 7, 10) *Low Partial Credit:* • correct factor(s) *High Partial Credit* • identification of equal terms
	Or $z-1-i\,\overline{)\,z^2-2z+iz+3-i}$ quotient $z-1+2i$ $\underline{z^2-z-iz}$ $-z+2iz+3-i$ $\underline{-z+1 \quad +\mathrm{i}}$ $2iz+2-2i$ $\underline{2iz+2-2i}$ $z-1+2i=0$ $z=1-2i$	**Or** Scale 10C (0, 3, 7, 10) *Low Partial Credit:* • long division formulated correctly *High Partial Credit* • two correct lines in division
	Or $(1+i)(m+ni) = 3-i$ $(m-n)+(m+n)i = 3+(-1)i$ $m-n=3 \quad and \quad m+n=-1$ Solving $\quad m=1 \quad and \quad n=-2$	**Or** Scale 10C (0, 3, 7, 10) *Low Partial Credit:* • correct multiplication • substitution of $(m+ni)$ into quadratic and stops *High Partial Credit* • identification of equal terms **Note:** *substitution of* $(1+i)$ *merits 0 marks*