Question 4 **(25 marks)**

(a) Prove by induction that $8^n - 1$ is divisible by 7 for all $n \in \mathbb{N}$.

(b) Given $\log_a 2 = p$ and $\log_a 3 = q$, where $a > 0$, write each of the following in terms of p and q:

(i) $\log_a \frac{8}{3}$

(ii) $\log_a \frac{9a^2}{16}$.

Q4	Model Solution – 25 Marks	Marking Notes
(a)	P_1: $8^1 - 1 = 7$ (divisible by 7) P_k: Assume $8^k - 1$ is divisible by 7 $8^k - 1 = 7M$ $8^k = 7M + 1$ P_{k+1}: $8^{k+1} - 1 = 8(8^k) - 1$ $= 8(7M + 1) - 1$ $= 56M + 7$ $= 7(8M + 1)$ P_{k+1} is divisible by 7 P_1 is true P_k true $\implies$ P_{k+1} is true So, P_{k+1} true whenever P_k true. Since P_1 true, then, by induction, P_n is true for all natural numbers ≥ 1 **Or** $P_{k+1} = 8^{k+1} - 1$ $= 8.8^k - 1$ $= (7 + 1).8^k - 1$ $= 7(8^k) + (8^k - 1)$ Obviously divisible by 7 From P_k So, P_{k+1} true whenever P_k true. Since P_1 true, then, by induction, P_n is true for all natural numbers ≥ 1	Scale 15D (0, 4, 7, 11, 15) *Low Partial Credit* • P_1 step *Mid Partial Credit* • P_k step • P_{k+1} step *High Partial Credit* • use of P_k step to prove P_{k+1} step **Note:** *accept* P_1 step, P_k step *and* P_{k+1} step *in any order*

(b) **(i)**	$p = \log_a 2\ ,\qquad q = \log_a 3$ $\log_a \frac{8}{3} = \log_a 8 - \log_a 3$ $= \log_a (2)^3 - \log_a 3$ $= 3\log_a 2 - \log_a 3$ $= 3p - q$	Scale 5C (0, 2, 4, 5) *low Partial Credit* • $\log_a 8 - \log_a 3$ *High Partial Credit* • $\log_a 8 = 3\log_a 2$ (and/or $= 3p$)
(ii)	$\log_a \frac{9a^2}{16} = \log_a (3a)^2 - \log_a (2)^4$ $= 2\log_a 3 + 2\log_a a - 4\log_a 2$ $= 2q + 2(1) - 4p$ $= 2q + 2 - 4p$	Scale 5D (0, 2, 3, 4, 5) *Low Partial Credit* • $\log_a 9a^2 - \log_a 16$ *Mid Partial Credit* • $2\log_a 3$ • $2\log_a a$ • $4\log_a 2$ • $4p$ or $2q$ or 2 *High Partial Credit* • $2(\log_a 3 + \log_a a) - 4\log_a 2$ or equivalent