Question 6 **(25 marks)**

(a) Differentiate the function $(2x+4)^2$ from first principles, with respect to x.

(b) **(i)** If $y = x\sin\left(\frac{1}{x}\right)$, find $\frac{dy}{dx}$.

(ii) Find the slope of the tangent to the curve $y = x\sin\left(\frac{1}{x}\right)$, when $x = \frac{4}{\pi}$.
Give your answer correct to two decimal places.

Q6	Model Solution – 25 Marks	Marking Notes
(a)	$f(x+h)-f(x)=(2x+2h+4)^2-(2x+4)^2$ $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=$ $\lim\limits_{h\to 0}\dfrac{(2x+2h+4)^2-(2x+4)^2}{h}$ $=\lim\limits_{h\to 0}\left(\dfrac{\begin{bmatrix}(4x^2+8hx+4h^2+16x+16h+16)\\-(4x^2+16x+16)\end{bmatrix}}{h}\right)$ $=\lim\limits_{h\to 0}\dfrac{8hx+4h^2+16h}{h}$ $=8x+16$ **or** $f(x)=(2x+4)^2=4x^2+16x+16$ $f(x+h)=4(x+h)^2+16(x+h)+16$ $=4x^2+8hx+4h^2+16x+16h+16$ $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $\lim\limits_{h\to 0}\dfrac{8hx+4h^2+16h}{h}$ $=8x+16$	Scale 10D (0, 2, 5, 8, 10) *Low Partial Credit* • any $f(x+h)$ *Mid Partial Credit* • limit of $\dfrac{f(x+h)-f(x)}{h}$ *High Partial Credit* • limit of $\dfrac{(2x+2h+4)^2-(2x+4)^2}{h}$ **Notes:** - *omission of limit sign penalised once only* - *answer not from 1st Principles merits 0 marks*

(b) **(i)+** **(ii)**	$$y = x.\sin\frac{1}{x}$$ $$\frac{dy}{dx} = \sin\frac{1}{x} + x\left(\cos\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)$$ $$\frac{dy}{dx} = \sin\frac{1}{x} - \frac{1}{x}\cos\frac{1}{x}$$ $$\frac{dy}{dx} = \sin\frac{\pi}{4} - \frac{\pi}{4}\cos\frac{\pi}{4}$$ $$= 0{\cdot}15$$	Scale 15D (0, 4, 7, 11, 15) *Low Partial Credit* • any correct differentiation *Mid Partial Credit* • product rule applied *High Partial Credit* • correct differentiation **Note:** *one penalty for calculator in wrong mode*