Question 7 **(40 marks)**

(a) **(i)** Air is pumped into a spherical exercise ball at the rate of 250 cm^3 per second. Find the rate at which the radius is increasing when the radius of the ball is 20 cm. Give your answer in terms of π.

(ii) Find the rate at which the surface area of the ball is increasing when the radius of the ball is 20 cm.

(b) The inflated ball is kicked into the air from a point *O* on the ground. Taking *O* as the origin, $(x,\ f(x))$ approximately describes the path followed by the ball in the air, where

$$f(x) = -x^2 + 10x$$

and both x and $f(x)$ are measured in metres.

(i) Find the values of x when the ball is on the ground.

(ii) Find the average height of the ball above the ground, during the interval from when it is kicked until it hits the ground again.

Q7	Model Solution – 40 Marks	Marking Notes
(a) **(i)**	$v = \frac{4}{3}\pi r^3 \Longrightarrow \frac{dv}{dr} = 4\pi r^2$ $\frac{dv}{dt} = 250\ \text{cm}^3/\text{s}$ $\frac{dr}{dt} = \frac{dr}{dv}.\frac{dv}{dt} = \frac{1}{4\pi r^2}.250$ $\frac{dr}{dt} = \frac{250}{4\pi 400} = \frac{5}{32\pi}\ \text{cm/s}$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • work towards $\frac{dv}{dr}$ or $\frac{dv}{dt}$ or $\frac{dr}{dt}$ *High Partial Credit* • correct expression for $\frac{dr}{dt}$
(ii)	$a = 4\pi r^2 \Longrightarrow \frac{da}{dr} = 8\pi r$ $\frac{da}{dt} = \frac{da}{dr}.\frac{dr}{dt} = 8\pi r.\frac{5}{32\pi}$ $= \frac{5(20)}{4}$ $= 25\ \text{cm}^2/\text{s}$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • work towards $\frac{da}{dr}$ or $\frac{da}{dt}$ *High Partial Credit* • correct expression for $\frac{da}{dt}$
(b) **(i)**	$-x^2 + 10x = 0$ $x(-x + 10) = 0$ $x = 0 \quad \text{or} \quad x = 10$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • quadratic equation formed • gets $x = 0$ only *High Partial Credit* • quadratic factorised **Note:** $f'(x) = 0 \Rightarrow 2x - 10 = 0 \Rightarrow x = 5$ *merits 0 marks*
(ii)	$\frac{1}{10-0}\int_0^{10}(-x^2 + 10x)dx$ $= \frac{1}{10}\left[\frac{-x^3}{3} + 5x^2\right]_0^{10}$ $= \frac{1}{10}\left[\left(\frac{-1000}{3} + 500\right) - 0\right]$ $= \frac{-100}{3} + 50 = \frac{50}{3}\text{m}$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • integration set up *High Partial Credit* • correct integration with some substitution