

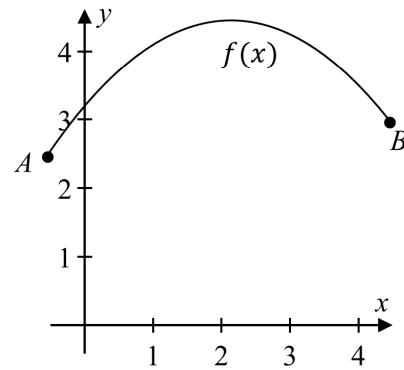
Question 8

(55 marks)

- (a) The diagram shows Sarah's first throw at the basket in a basketball game. The ball left her hands at A and entered the basket at B . Using the co-ordinate plane with $A(-0.5, 2.565)$ and $B(4.5, 3.05)$, the equation of the path of the centre of the ball is

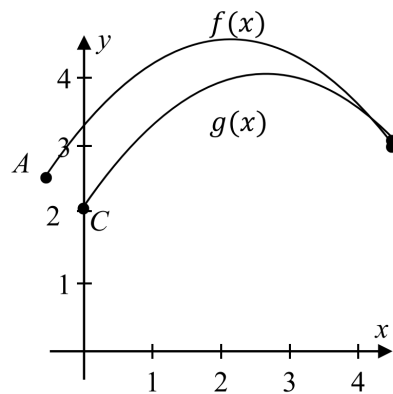
$$f(x) = -0.274x^2 + 1.193x + 3.23,$$

where both x and $f(x)$ are measured in metres.



- (i) Find the maximum height reached by the centre of the ball, correct to three decimal places.
- (ii) Find the acute angle to the horizontal at which the ball entered the basket. Give your answer correct to the nearest degree.

- (iii) Sarah took a second throw. This throw followed the path of the parabola $g(x)$ as shown. The ball left Sarah's hands at the point $C(0, 2)$. The graph $y = g(x)$ is the image of the graph $y = f(x)$ under the translation which maps A onto C . Using your result from part **a(i)**, show that the centre of this ball reached its maximum height at the point $(2.677, 3.964)$, correct to three decimal places.



- (iv) Hence, or otherwise, find the equation of the parabola $g(x)$.
- (b) The heptathlon is an Olympic competition. It consists of seven events including the 200 m race and the javelin. The scoring system uses formulas to calculate a score for each event. The table below shows the formulas for two of the events and the values of constants used in these formulas, where x is the time taken (in seconds) or distance achieved (in metres) by the competitor and y is the number of points scored in the event.

Event	x	Formula	a	b	c
200 m race	Time (s)	$y = a(b - x)^c$	4.99087	42.5	1.81
Javelin	Distance (m)	$y = a(x - b)^c$	15.9803	3.8	1.04

- (i) In the heptathlon, Jessica ran 200 m in 23.8 s and threw the javelin 58.2 m. Use the formulas in the table to find the number of points she scored in each of these events, correct to the nearest point.
- (ii) The world record distance for the javelin, in the heptathlon, would merit a score of 1295 points. Find the world record distance for the javelin, in the heptathlon, correct to two decimal places.
- (iii) The formula used to calculate the points for the 800 m race, in the heptathlon, is the same formula used for the 200 m race but with different constants. Jessica ran the 800 m race in 2 minutes and 1.84 seconds which merited 1087 points. If $a = 0.11193$ and $b = 254$ for the 800 m race, find the value of c for this event, correct to two decimal places.

Q8	Model Solution – 55 Marks	Marking Notes
(a) (i)	$f(x) = -0.274x^2 + 1.193x + 3.23$ $f'(x) = -0.548x + 1.193 = 0$ $x = 2.177 \text{ m}$ $f(2.177) = -0.274(2.177)^2 + 1.193(2.177) + 3.23$ $= -1.2986 + 2.5972 + 3.23$ $= 4.529 \text{ m}$ <p style="text-align: center;"><i>or</i></p> $-0.274\left(x^2 - \frac{1193}{274}x - \frac{1615}{137}\right)$ $-0.274\left(x - \frac{1193}{548}\right)^2 + 4.5285$ $\text{Max Height} = 4.529 \text{ m}$	<p>Scale 10C (0, 3, 7, 10)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> any correct differentiation effort made at completing square trial and error with more than one value of x tested <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> x value correct <p>Note: if correct answer by trial and error, must show points on each side of max point to be lower to earn full credit</p>
(ii)	$\tan \theta = -0.548(4.5) + 1.193$ $\tan \theta = -1.273$ $\theta = 51.8^\circ = 52^\circ$	<p>Scale 5B (0, 2, 5)</p> <p><i>Partial Credit</i></p> <ul style="list-style-type: none"> \tan <p>Note: right angled triangles may appear in diagram given in equation</p>
(iii)	<p>Map $A \rightarrow C$</p> $(-0.5, 2.565) \rightarrow (0, 2)$ $2.177 - (-0.5) = 2.677$ $4.529 - 0.565 = 3.964$ $(2.177, 4.529) \rightarrow (2.677, 3.964)$	<p>Scale 5B (0, 2, 5)</p> <p><i>Partial Credit</i></p> <ul style="list-style-type: none"> $(-0.5, 2.565) \rightarrow (0, 2)$

<p>(iv)</p>	$g(x) = ax^2 + bx + c$ $C(0, 2) \in g(x) \Rightarrow c = 2$ $B(4.5, 3.05) \in g(x)$ $3.05 = a(4.5)^2 + b(4.5) + 2$ $\Rightarrow 20.25a + 4.5b = 1.05 \quad \dots \text{(i)}$ $g'(x) = 2ax + b = 0$ $\Rightarrow 2a(2.677) + b = 0$ $5.354a + b = 0 \quad \dots \text{(ii)}$ <p>From (i) and (ii)</p> $a = -0.273$ $b = 1.462$ $g(x) = -0.273x^2 + 1.462x + 2$ <p>[Note: a third equation that could be used is</p> $3.964 = a(2.677)^2 + b(2.677) + 2 \quad \dots \text{(iii)}$ <p style="text-align: center;">Or</p> <p>Equation of parabola with vertex (h, k):</p> $g(x) = a(x - h)^2 + k$ $C(0, 2) \text{ on curve: } (h, k) = (2.677, 3.964)$ $2 = a(-2.677)^2 + 3.964$ $-1.964 = a(7.166329)$ $a = -0.27405 = -0.274$ <p>Parabola:</p> $g(x) = -0.274[(x - 2.677)^2] + 3.964$ <p style="text-align: center;">or</p> $g(x) = f(x - 0.5) - 0.565$ $g(x) = -0.274(x - 0.5)^2 + 1.193(x - 0.5) + 3.23 - 0.565$ $g(x) = -0.274x^2 + 1.467x + 2$	<p>Scale 10D (0, 2, 5, 8, 10)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> • c value found • relevant equation in a, b and/or c <p><i>Mid Partial Credit</i></p> <ul style="list-style-type: none"> • formulated correctly any two equations <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> • formulated correctly any three equations <p>Note: $ax^2 + bx + c$ not in an equation merits 0 marks</p> <p style="text-align: center;">Or</p> <p>Scale 10D (0, 2, 5, 8, 10)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> • equation of curve • use of C <p><i>Mid Partial Credit</i></p> <ul style="list-style-type: none"> • using peak value <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> • value of a found
--------------------	--	--

<p>(b) (i)</p>	<p>200 m Race:</p> $y = a(b - x)^c$ $y = 4.99087(42.5 - 23.8)^{1.81}$ $y = 1000$ <p>Javelin:</p> $y = a(x - b)^c$ $y = 15.9803(58.2 - 3.8)^{1.04}$ $y = 1020$	<p>Scale 10D (0, 2, 5, 8, 10)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> • some relevant substitution into one formula <p><i>Mid Partial Credit</i></p> <ul style="list-style-type: none"> • one value of y found • some relevant substitution into both formulas <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> • one value correct and some relevant substitution into second formula • uses incorrect formula (once only)
<p>(ii)</p>	$y = a(x - b)^c$ $1295 = 15.9803(x - 3.8)^{1.04}$ $81.0373 = (x - 3.8)^{1.04} = z^{1.04}$ $\log z = \frac{\log 81.0373}{1.04}$ $z = 68.4343 = (x - 3.8)$ $x = 72.2343 = 72.23 \text{ m}$	<p>Scale 5B (0, 2, 5)</p> <p><i>Partial Credit</i></p> <ul style="list-style-type: none"> • some relevant substitution into formula
<p>(iii)</p>	$y = a(b - x)^c$ $1087 = 0.11193(254 - 121.84)^c$ $\frac{1087}{0.11193} = (132.16)^c$ $\log 9711.426 = c \log 132.16$ $c = \frac{\log 9711.426}{\log 132.16} = 1.88$	<p>Scale 10C (0, 3, 7, 10)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> • some relevant substitution into formula <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> • fully correct substitution into formula