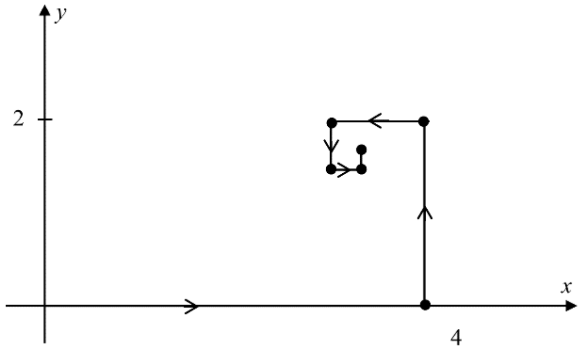
Question 9 **(55 marks)**

(a) At the first stage of a pattern, a point moves 4 units from the origin in the positive direction along the x-axis. For the second stage, it turns left and moves 2 units parallel to the y-axis. For the third stage, it turns left and moves 1 unit parallel to the x-axis.
At each stage, after the first one, the point turns left and moves half the distance of the previous stage, as shown.



(i) How many stages has the point completed when the total distance it has travelled, along its path, is 7·9375 units?

(ii) Find the maximum distance the point can move, along its path, if it continues in this pattern indefinitely.

(iii) Complete the second row of the table below showing the changes to the x co-ordinate, the first nine times the point moves to a new position. Hence, or otherwise, find the x co-ordinate and the y co-ordinate of the final position that the point is approaching, if it continues indefinitely in this pattern.

Stage	**1st**	**2nd**	**3rd**	**4th**	**5th**	**6th**	**7th**	**8th**	**9th**
Change in x	+4	0	−1						
Change in y									

(b) A male bee comes from an unfertilised egg, i.e. he has a female parent but he does not have a male parent. A female bee comes from a fertilised egg, i.e. she has a female parent and a male parent.

(i) The following diagram shows the ancestors of a certain male bee. We identify his generation as G_1 and our diagram goes back to G_4. Continue the diagram to G_5.

G_1	G_2	G_3	G_4	G_5
			Female	
		Female		
Male	Female		Male	
		Male	Female	

(ii) The number of ancestors of this bee in each generation can be calculated by the formula

$$G_{n+2} = G_{n+1} + G_n,$$

where $G_1 = 1$ and $G_2 = 1$, as in the diagram.
Use this formula to calculate the number of ancestors in G_6 and in G_7.

(iii) The number of ancestors in each generation can also be calculated by using the formula

$$G_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}.$$

Use this formula to verify the number of ancestors in G_3.

Q9	Model Solution – 55 Marks	Marking Notes
(a)(i)	$4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ $S_n = \frac{a(1-r^n)}{1-r}$ $S_n = \frac{4\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}} = 7{\cdot}9375$ $-\frac{1}{2^n} = -\frac{1}{128}$ $n = 7$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • some listing of terms • S_n formula *High Partial Credit* • listing of exactly 7 correct terms • formula fully substituted
(a) **(ii)**	$S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{4}{1-\frac{1}{2}} = 8$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • S_∞ formula *High Partial Credit* • formula fully substituted

	1	2	3	4	5	6	7	8	9
Chg x	+4	0	-1	0	$\frac{1}{4}$	0	$-\frac{1}{16}$	0	$\frac{1}{64}$
Chg y	0	2	0	$-\frac{1}{2}$	0	$\frac{1}{8}$	0	$-\frac{1}{32}$	0

(a) (iii)	$S_\infty = \frac{4}{1-\left(-\frac{1}{4}\right)} = 3{\cdot}2 = \frac{16}{5}$ $S_\infty = \frac{2}{1-\left(-\frac{1}{4}\right)} = 1{\cdot}6 = \frac{8}{5}$ $\left(\frac{16}{5}, \frac{8}{5}\right)$ or $(3{\cdot}2, 1{\cdot}6)$	Scale 15D (0, 4, 7, 11, 15) *Low Partial Credit* • 2 extra entries correct in either row *Mid Partial Credit* • either row fully correct *High Partial Credit* • one co-ordinate correct **Notes:** - *need to see S_∞ correctly used to move beyond Mid Partial Credit* - *no S_∞ merits Mid Partial Credit at most*
(b) (i)	G_5 =Female,Male,Female,Female,Male	Scale 5B (0, 2, 5) *Partial Credit* • one correct entry
(b) (ii)	$G_6 = G_5 + G_4 = 5 + 3 = 8$ $G_7 = G_6 + G_5 = 8 + 5 = 13$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • $G_6 = G_5 + G_4$ • $G_7 = G_6 + G_5$ • G_7 or G_6 correct • 8 and/or 13 without work *High Partial Credit* • correct substitution in both

(b) (iii)	$G_3 = \frac{(1+\sqrt{5})^3 - (1-\sqrt{5})^3}{2^3\sqrt{5}} = 2$ $(1+\sqrt{5})^3 = \left(1 + 3\sqrt{5} + 3\sqrt{5}^2 + \sqrt{5}^3\right)$ $= 16 + 8\sqrt{5}$ $(1-\sqrt{5})^3 = \left(1 - 3\sqrt{5} + 3\sqrt{5}^2 - \sqrt{5}^3\right)$ $= 16 - 8\sqrt{5}$ $G_3 = \frac{6\sqrt{5} + 2\sqrt{5}^3}{8\sqrt{5}}$ $= \frac{6 + 2\sqrt{5}^2}{8} = \frac{16}{8} = 2 \quad \text{Q.E.D.}$	Scale 5B (0, 2, 5) *Partial Credit* • some correct substitution • using approximate value for $\sqrt{5}$ • $G_3 = 2$ • some effort at cubing **Note:** *use of* $\sqrt{5}$ *as approximation, even if rounded off to* 2 *at end of work merits at most Partial Credit*