**Question 3** **(25 marks)**

**(a)** Show that $\dfrac{\cos 7A + \cos A}{\sin 7A - \sin A} = \cot 3A$.

**(b)** Given that $\cos 2\theta = \frac{1}{9}$, find $\cos\theta$ in the form $\pm\frac{\sqrt{a}}{b}$, where $a, b \in \mathbb{N}$.

| Q3 | Model Solution – 25 Marks | Marking Notes |
|---|---|---|
| (a) | $\dfrac{2\cos\frac{7A+A}{2}\cos\frac{7A-A}{2}}{2\cos\frac{7A+A}{2}\sin\frac{7A-A}{2}}$<br>$\dfrac{2\cos 4A\cos 3A}{2\cos 4A\sin 3A}$<br>$=\dfrac{\cos 3A}{\sin 3A}$<br>$=\cot 3A$ | Scale 15C (0, 5, 10, 15)<br>*Low Partial Credit*<br>• sum to product formula with some substitution<br>*High Partial Credit*<br>• sum to product formula fully substituted |
| (b) | Method 1:<br>$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$<br>$=\frac{1}{2}\left(1+\frac{1}{9}\right)=\frac{5}{9}$<br>$\cos\theta = \pm\frac{\sqrt{5}}{3}$<br>**or**<br>Method 2:<br>$\cos 2\theta = 1 - 2\sin\theta = \frac{1}{9}$<br>$9 - 18\sin^2\theta = 1$<br>$\sin^2\theta = \frac{4}{9} \Rightarrow \sin\theta = \pm\frac{2}{3} \Rightarrow \cos\theta = \pm\frac{\sqrt{5}}{3}$<br>**or**<br>Method 3:<br>$\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1}{9}$<br>$9 - 9\tan^2\theta = 1 + \tan^2\theta$<br>$\tan^2\theta = \frac{4}{5}$<br>$\Rightarrow \tan\theta = \pm\frac{2}{\sqrt{5}} \Rightarrow \cos\theta = \pm\frac{\sqrt{5}}{3}$ | Scale 10D (0, 3, 5, 8, 10)<br>*Low Partial Credit*<br>• Use of a relevant formula in $\cos 2\theta$<br>• $\cos^{-1}\left(\frac{1}{9}\right) = 83{\cdot}62°$<br>• $\theta = 41{\cdot}8°$<br>*Mid Partial Credit*<br>• correct substitution (method 1)<br>• expression in $\sin^2\theta$ (method 2)<br>• expression in $\tan^2\theta$ (method 3)<br>• expression in $\cos^2\theta$ (method 4)<br>• $\theta = 41{\cdot}8°$ and $\theta = 132{\cdot}2°$ or $\theta = 221{\cdot}8°$<br>*High Partial Credit*<br>• one value only (e.g. $+\frac{\sqrt{5}}{3}$)<br>• values found for cos 41·8° and cos 138·2° or cos 221·8° |

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| | **or**<br>Method 4:<br>$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$<br>$1-\cos^2\theta = \frac{1}{2}(1-\cos 2\theta)$<br>$2-2\cos^2\theta = 1-\cos 2\theta$<br>$\cos^2\theta = \frac{1+\cos 2\theta}{2} = \frac{1+\frac{1}{9}}{2}$<br>$\cos^2\theta = \frac{5}{9}$<br>$\cos\theta = \pm\frac{\sqrt{5}}{3}$ | |