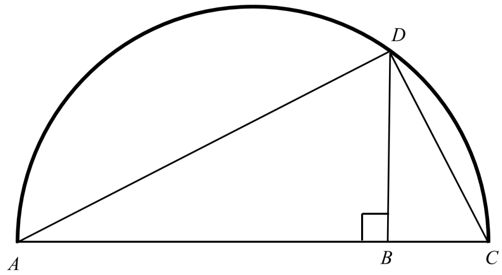
**Question 4** **(25 marks)**

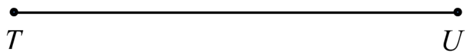
The diagram shows a semi-circle standing on a diameter $[AC]$, and $[BD] \perp [AC]$.

**(a)** **(i)** Prove that the triangles *ABD* and *DBC* are similar.

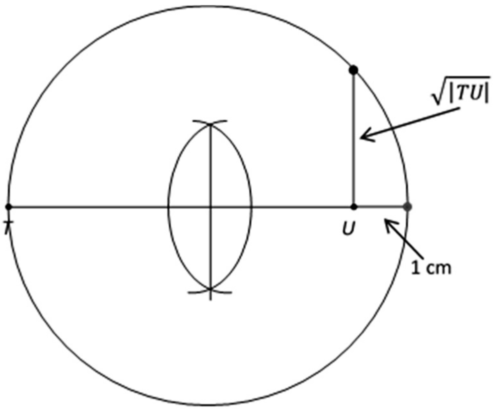



**(ii)** If $|AB| = x$, $|BC| = 1$, and $|BD| = y$, write $y$ in terms of $x$.

**(b)** Use your result from part **(a)(ii)** to **construct** a line segment equal in length (in centimetres) to the square root of the length of the line segment $[TU]$ which is drawn below.



| Q4 | Model Solution – 25 Marks | Marking Notes |
|---|---|---|
| **(a)** **(i)** | $\lvert\angle ABD\rvert = \lvert\angle CBD\rvert = 90°$…….(i)<br>$\lvert\angle BDC\rvert + \lvert\angle BCD\rvert = 90°$ …*angles in triangle sum to* $180°$<br>$\lvert\angle ADB\rvert + \lvert\angle BDC\rvert = 90°$ …. *angle in semicircle*<br>$\lvert\angle ADB\rvert + \lvert\angle BDC\rvert = \lvert\angle BDC\rvert + \lvert\angle BCD\rvert$<br>$\lvert\angle ADB\rvert = \lvert\angle BCD\rvert$……..(ii)<br>$\therefore$ *Triangles are equiangular (or similar)*<br>**or**<br>$\lvert\angle ABD\rvert = \lvert\angle CBD\rvert = 90°$…….(i)<br>$\lvert\angle DAB\rvert = \lvert\angle DAC\rvert$ *same angle* $\Rightarrow \lvert\angle ADB\rvert = \lvert\angle DCA\rvert$ *(reasons as above) which is also* $\angle DCB$………(ii) | Scale 15C (0, 5, 10, 15)<br>*Low Partial Credit*<br>• identifies one angle of same size in each triangle<br><br>*High Partial Credit*<br>• identifies second angle of same size in each triangle<br>• implies triangles are similar without justifying (ii) in model solution or equivalent |
| **(a)** **(ii)** | $\frac{y}{1} = \frac{x}{y}$<br>$\Rightarrow y^2 = x$<br>$y = \sqrt{x}$<br>**or**<br>$\lvert AD\rvert^2 + \lvert DC\rvert^2 = \lvert AC\rvert^2$<br>$\lvert AD\rvert = \sqrt{x^2 + y^2}$<br>$\lvert DC\rvert = \sqrt{y^2 + 1}$<br>$x^2 + y^2 + y^2 + 1 = (x+1)^2$<br>$2y^2 = 2x$<br>$y = \sqrt{x}$<br>**Or**<br>$\frac{\sqrt{x^2+y^2}}{\sqrt{y^2+1}} = \frac{y}{1} \Rightarrow x^2 + y^2 = y^2(y^2+1)$<br>$y^4 = x^2 \Rightarrow y^2 = x \Rightarrow y = \sqrt{x}$ | Scale 5C (0, 2, 4, 5)<br>*Low Partial Credit*<br>• one set of corresponding sides identified<br>• indicates relevant use of Pythagoras<br><br>*High Partial Credit*<br>• corresponding sides fully substituted<br>• expression in $y^2$ or $y^4$, i.e. fails to finish |

| | | |
|---|---|---|
| **(b)** | Construction<br><br> | Scale 5C (0, 2, 4, 5)<br>*Low Partial Credit*<br>• perpendicular line drawn at *U* or *T*<br>• relevant use of 1 cm length<br>• mid point of incorrect extended segment constructed<br><br>*High Partial Credit*<br>• correct mid-point constructed |