

Question 6**(25 marks)**

A local sports club is planning to run a weekly lotto. To win the Jackpot of €1000, contestants must match one letter chosen from the 26 letters in the alphabet and two numbers chosen, in the correct order, from the numbers 0 to 9. In this lotto, repetition of numbers is allowed (e.g. M, 3, 3 is an outcome).

- (a) Calculate the probability that M, 3, 3 would be the winning outcome in a particular week.
- (b) If a contestant matches the letter only, or the letter and one number (but not both numbers), they will win €50. Using the table below, or otherwise, find how much the club should expect to make or lose on each play, correct to the nearest cent, if they charge €2 per play.

Event	Payout (x) €	Probability ($P(x)$)	$x.P(x)$
Win Jackpot			
Match letter and first number only			
Match letter and second number only			
Match letter and neither number			
Fail to win			

- (c) The club estimates that the average number of plays per week will be 845. If the club wants to make an average profit of €600 per week from the lotto, how much should the club charge per play, correct to the nearest cent?

Q6	Model Solution – 25 Marks				Marking Notes																																																
(a)	$P(M, 3, 3) = \frac{1}{26} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{2600}$				<p>Scale 10C (0, 3, 7, 10)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> any correct relevant probability <p><i>High Partial credit</i></p> <ul style="list-style-type: none"> correct probabilities but not expressed as single fraction or equivalent <p>Note: Accept correct answer without supporting work</p>																																																
(b)	<table border="1" data-bbox="248 645 823 1104"> <thead> <tr> <th>Event</th> <th>Payout</th> <th>Prob (P(x))</th> <th>x.P(x)</th> </tr> </thead> <tbody> <tr> <td>Win</td> <td>1000</td> <td>$\frac{1}{2600}$</td> <td>$\frac{1000}{2600}$</td> </tr> <tr> <td>letter 1 No.</td> <td>50</td> <td>$\frac{9}{2600}$</td> <td>$\frac{450}{2600}$</td> </tr> <tr> <td>letter 2nd No</td> <td>50</td> <td>$\frac{9}{2600}$</td> <td>$\frac{450}{2600}$</td> </tr> <tr> <td>letter only</td> <td>50</td> <td>$\frac{81}{2600}$</td> <td>$\frac{4050}{2600}$</td> </tr> <tr> <td>Fail to win</td> <td>0</td> <td></td> <td>0</td> </tr> </tbody> </table> $\sum x.P(x) = \frac{5950}{2600} = 2.29$ <p>Club loses 29 cent per play</p> <p style="text-align: center;">Or</p> <table border="1" data-bbox="248 1346 836 1823"> <thead> <tr> <th>Event</th> <th>Pay out</th> <th>Prob (P(x))</th> <th>x.P(x)</th> </tr> </thead> <tbody> <tr> <td>Win</td> <td>-998</td> <td>$\frac{1}{2600}$</td> <td>$-\frac{998}{2600}$</td> </tr> <tr> <td>letter + 1st No.</td> <td>-48</td> <td>$\frac{9}{2600}$</td> <td>$-\frac{432}{2600}$</td> </tr> <tr> <td>Letter + 2nd No</td> <td>-48</td> <td>$\frac{9}{2600}$</td> <td>$-\frac{432}{2600}$</td> </tr> <tr> <td>letter only</td> <td>-48</td> <td>$\frac{81}{2600}$</td> <td>$-\frac{3888}{2600}$</td> </tr> <tr> <td>Fail to Win</td> <td>+2</td> <td>$\frac{2500}{2600}$</td> <td>$\frac{5000}{2600}$</td> </tr> </tbody> </table> $\sum x.P(x) = -\frac{750}{2600} = -29 \text{ cent}$				Event	Payout	Prob (P(x))	x.P(x)	Win	1000	$\frac{1}{2600}$	$\frac{1000}{2600}$	letter 1 No.	50	$\frac{9}{2600}$	$\frac{450}{2600}$	letter 2 nd No	50	$\frac{9}{2600}$	$\frac{450}{2600}$	letter only	50	$\frac{81}{2600}$	$\frac{4050}{2600}$	Fail to win	0		0	Event	Pay out	Prob (P(x))	x.P(x)	Win	-998	$\frac{1}{2600}$	$-\frac{998}{2600}$	letter + 1 st No.	-48	$\frac{9}{2600}$	$-\frac{432}{2600}$	Letter + 2 nd No	-48	$\frac{9}{2600}$	$-\frac{432}{2600}$	letter only	-48	$\frac{81}{2600}$	$-\frac{3888}{2600}$	Fail to Win	+2	$\frac{2500}{2600}$	$\frac{5000}{2600}$	<p>Scale 10C (0, 3, 7, 10)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> 1 correct entry to table <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> all entries correct but fails to finish or finishes incorrectly no conclusion
Event	Payout	Prob (P(x))	x.P(x)																																																		
Win	1000	$\frac{1}{2600}$	$\frac{1000}{2600}$																																																		
letter 1 No.	50	$\frac{9}{2600}$	$\frac{450}{2600}$																																																		
letter 2 nd No	50	$\frac{9}{2600}$	$\frac{450}{2600}$																																																		
letter only	50	$\frac{81}{2600}$	$\frac{4050}{2600}$																																																		
Fail to win	0		0																																																		
Event	Pay out	Prob (P(x))	x.P(x)																																																		
Win	-998	$\frac{1}{2600}$	$-\frac{998}{2600}$																																																		
letter + 1 st No.	-48	$\frac{9}{2600}$	$-\frac{432}{2600}$																																																		
Letter + 2 nd No	-48	$\frac{9}{2600}$	$-\frac{432}{2600}$																																																		
letter only	-48	$\frac{81}{2600}$	$-\frac{3888}{2600}$																																																		
Fail to Win	+2	$\frac{2500}{2600}$	$\frac{5000}{2600}$																																																		

(c)	<p>Profit = Revenue – Pay-out</p> $600 = 845(x - 2.29)$ $x = \frac{600 + 845(2.29)}{845}$ $x = 3$ <p>or</p> $\frac{600}{845} = 0.71$ $0.71 + 2.29 = 3$	<p>Scale 5C (0, 2, 4, 5)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> • links profit, revenue and payout <p><i>High partial Credit</i></p> <ul style="list-style-type: none"> • formula fully substituted
-----	---	--