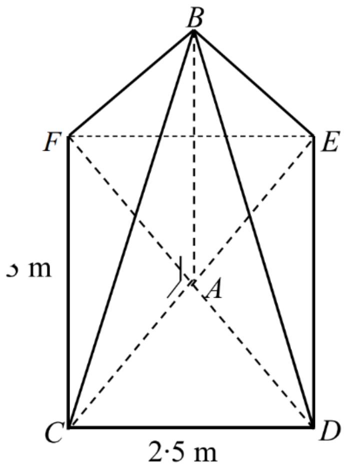
Question 7 **(55 marks)**

A glass Roof Lantern in the shape of a pyramid has a rectangular base *CDEF* and its apex is at *B* as shown. The vertical height of the pyramid is $|AB|$, where *A* is the point of intersection of the diagonals of the base as shown in the diagram.
Also $|CD| = 2{\cdot}5$ m and $|CF| = 3$ m.



(a) **(i)** Show that $|AC| = 1{\cdot}95$ m, correct to two decimal places.

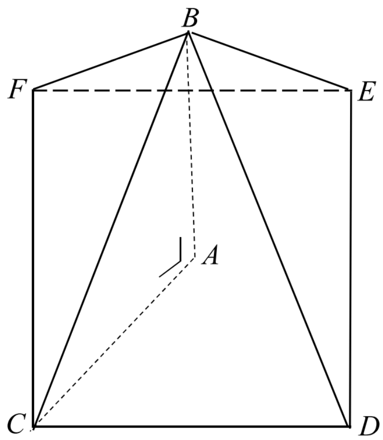
(ii) The angle of elevation of *B* from *C* is 50° (i.e. $|\angle BCA| = 50°$). Show that $|AB| = 2{\cdot}3$ m, correct to one decimal place.

(iii) Find $|BC|$, correct to the nearest metre.

(iv) Find $|\angle BCD|$, correct to the nearest degree.

(v) Find the area of glass required to glaze all four triangular sides of the pyramid. Give your answer correct to the nearest m^2.

(b) Another Roof Lantern, in the shape of a pyramid, has a square base *CDEF*. The vertical height $|AB| = 3$ m, where *A* is the point of intersection of the diagonals of the base as shown.
The angle of elevation of *B* from *C* is 60°
(i.e. $|\angle BCA| = 60°$).
Find the length of the side of the square base of the lantern.
Give your answer in the form $\sqrt{a}$ m, where $a \in \mathbb{N}$.



Q7	Model Solution – 55 Marks	Marking Notes
(a) (i)	$\|EC\|^2 = 3^2 + 2{\cdot}5^2 = 15{\cdot}25$ $\|EC\| = \sqrt{15{\cdot}25}$ $\|EC\| = 3{\cdot}905$ $\Rightarrow \|AC\| = 1{\cdot}9525$ $= 1{\cdot}95$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • Pythagoras with relevant substitution *High Partial Credit* • $\|EC\|$ correct • $\|AC\| = \frac{1}{2}\sqrt{15{\cdot}25}$
(a) (ii)	$\tan 50° = \frac{\|AB\|}{1{\cdot}95}$ $\|AB\| = 1{\cdot}95(1{\cdot}19175) = 2{\cdot}23239$ $\|AB\| = 2{\cdot}3$	Scale 10B (0, 5, 10) *Partial Credit* • tan formulated correctly
(a) (iii)	$\|BC\|^2 = 1{\cdot}95^2 + 2{\cdot}3^2$ $\|BC\| = 3 \cdot 015377$ $\|BC\| = 3$ Also: $\sin 40° = \frac{1{\cdot}95}{\|BC\|}$ **or** $\cos 40° = \frac{2{\cdot}3}{\|BC\|}$ **or** $\cos 50° = \frac{1{\cdot}95}{\|BC\|}$ **or** $\sin 50° = \frac{2{\cdot}3}{\|BC\|}$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • Pythagoras with relevant substitution *High Partial Credit* • Pythagoras fully substituted • $\|BC\| = \frac{1{\cdot}95}{\sin 40°}$ (i.e. $\|BC\|$isolated)
(a) (iv)	$3^2 = 3^2 + 2{\cdot}5^2 - 2(3)(2{\cdot}5)\cos \propto$ $15 \cos \propto = 6{\cdot}25$ $\propto = 65°$ **or** $\cos \propto = \frac{1{\cdot}25}{3}$ $\propto = 65°$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • cosine rule with some relevant substitution • cosine ratio with some relevant substitutions • identifies three sides of triangle *BCD* *High Partial Credit* • cosine rule with full relevant substitutions • cosine ratio with full relevant substitutions

(a) **(v)**	$A = 2\times$ isosceles triangle $+ 2\times$ equilateral triangle $= 2\times\left[\frac{1}{2}(2\cdot5)(3)\sin 65°\right] +$ $2\times\left[\frac{1}{2}(3)(3)\sin 60°\right]$ $= 14\cdot59$ $A = 15$	Scale 10D (0,3,5,8,10) *Low Partial Credit* • recognises area of 4 triangles *Mid Partial Credit* • Area of 1 triangle correct *High Partial Credit* • area of isosceles triangle and equilateral triangle **Note**: Area = 4 isosceles or 4 equilateral triangles merit *HPC* at most
(b)	$\tan 60° = \frac{3}{\lvert CA\rvert}$ $\Rightarrow \lvert CA\rvert = \sqrt{3}$ $\lvert CE\rvert = 2\sqrt{3}$ $x^2 + x^2 = \left(2\sqrt{3}\right)^2$ $x = \sqrt{6}$	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • effort at Pythagoras but without $\lvert CA\rvert$ (or $\lvert CE\rvert$) • $\lvert CA\rvert$ found *High Partial Credit* • $\lvert CE\rvert = 2\sqrt{3}$