Question 8 **(45 marks)**

The height of the water in a port was measured over a period of time. The average height was found to be 1·6 m. The height measured in metres, $h(t)$, was modelled using the function

$$h(t) = 1{\cdot}6 + 1{\cdot}5\cos\left(\frac{\pi}{6}t\right)$$

where t represents the number of hours since the last recorded high tide and $\left(\frac{\pi}{6}t\right)$ is expressed in radians.

(a) Find the period and range of $h(t)$.

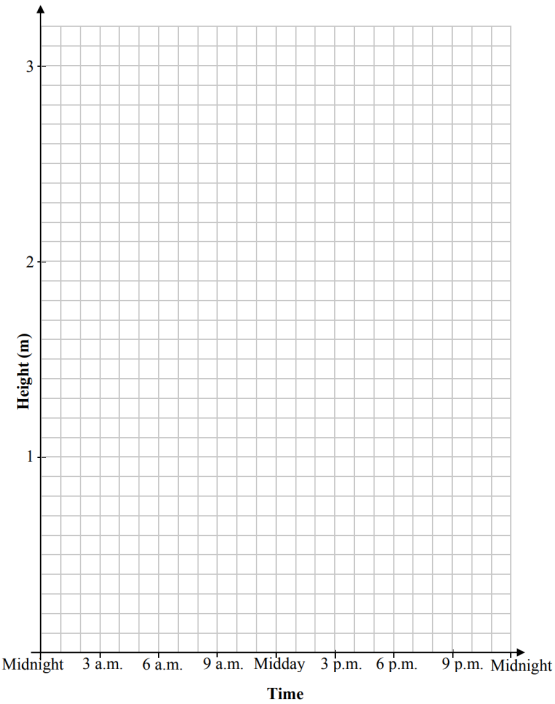
(b) Find the maximum height of the water in the port.

(c) Find the rate at which the height of the water is changing when $t = 2$, correct to two decimal places. Explain your answer in the context of the question.

(d) (i) On a particular day the high tide occurred at midnight (i.e. $t = 0$). Use the function to complete the table and show the height, $h(t)$, of the water between midnight and the following midnight.

$h(t) = 1{\cdot}6 + 1{\cdot}5\cos\left(\frac{\pi}{6}t\right)$									
Time	Midnight	3 a.m.	6 a.m.	9 a.m.	12 noon	3 p.m.	6 p.m.	9 p.m.	Midnight
t (hours)	0	3							
$h(t)$ (m)									

(ii) Sketch the graph of $h(t)$ between midnight and the following midnight.



(e) Find, from your sketch, the difference in water height between low tide and high tide.

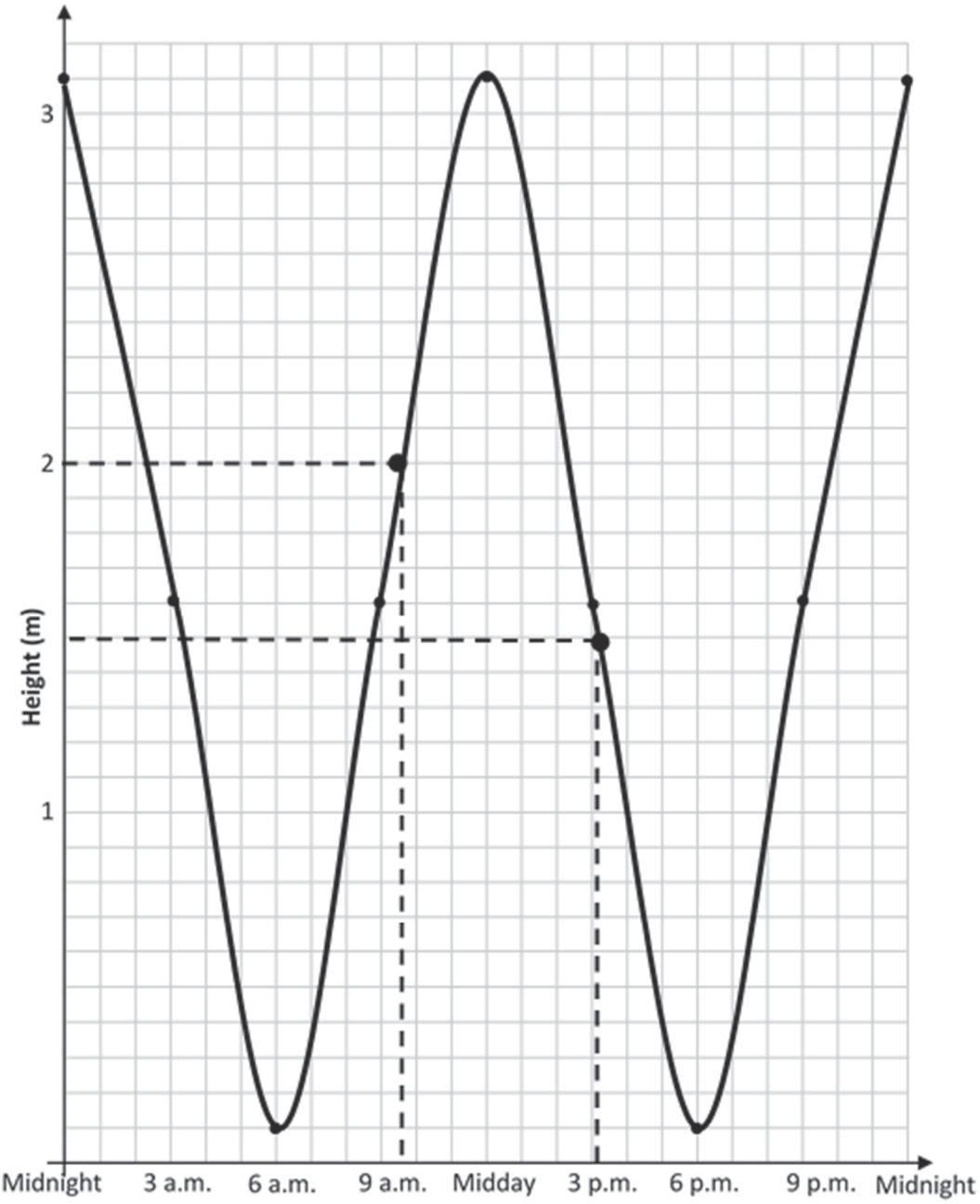
(f) A fully loaded barge enters the port, unloads its cargo and departs some time later.
The fully loaded barge requires a minimum water level of 2 m.
When the barge is unloaded it only requires 1·5 m.
Use your graph to estimate the **maximum** amount of time that the barge can spend in port, without resting on the sea-bed.

Q8	Model Solution – 45 Marks	Marking Notes
(a)	Period = $\frac{2\pi}{\frac{\pi}{6}}$ = 12 hours Range = $[1{\cdot}6 - 1{\cdot}5\ ,1{\cdot}6 + 1{\cdot}5\ ] = [0{\cdot}1\ \text{m}, 3{\cdot}1\ \text{m}]$	Scale 5C (0, 2,4, 5) *Low Partial Credit* • some use of 2π or $\frac{\pi}{6}$ • range of cos function *High partial credit* • period or range correct **Note:** *Accept correct period and/or range without work*
(b)	Max = 1·6 + 1·5(1) =3·1 m. **or** 3·1 m from range	Scale 5B (0,2, 5) *Partial Credit* • max occurs when cos *A* = 1 or *t* = 0 • effort at $h'(t)$ **Note:** *Accept correct answer without work*
(c)	$h'(t) = 1{\cdot}5(-\sin\frac{\pi t}{6})\frac{\pi}{6}$ $h'(2) = 1{\cdot}5(-\sin\frac{2\pi}{6})\frac{\pi}{6}$ $= -0{\cdot}68017 = -0{\cdot}68$ m/h Tide is going out at a rate of $0{\cdot}68$ m per hour at 2 am	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • effort at differentiation *High Partial Credit* • correct numerical answer but not in context

(d)(i)

$h(t) = 1 \cdot 6 + 1 \cdot 5 \cos\left(\frac{\pi}{6}t\right)$									
Time	12 am	3 am	6 am	9 am	12 pm	3 pm	6 pm	9 pm	12 am
t	0	3	6	9	12	15	18	21	24
Height	3·1	1·6	·1	1·6	3·1	1·6	·1	1·6	3·1

(d) **(i)**		Scale 10C (0, 3, 7, 10) *Low Partial Credit* • one correct height *High Partial Credit* • five correct heights

(d) **(ii)**		
(d) **(ii)**	Graph	Scale 10C (0, 3,7,10) *Low Partial Credit* • one correct plot *High Partial Credit* • at least 7 correct plots • plots correct but graph not sketched or sketched incorrectly

(e)	Low tide = $0{\cdot}1$ m High tide = $3{\cdot}1$ m Difference = $3{\cdot}1 - 0{\cdot}1 = 3$ m	Scale 5B (0, 2, 5) *Partial Credit* • height of Low tide or High tide correctly identified **Notes:** (i) *candidates may show work for this section on graph* (ii) *accept values from candidate's graph* (iii) *accept correct answer from graph without work*
(f)	Enter port at 9:30 approx Leave port before 15:15 approx Time = 15:15 – 9:30 = 5 hr 45 min approx.	Scale 5B (0, 2, 5) *Partial Credit* • time of entry to port or leave port correctly identified • value(s) for $h = 2$ and/or $h = 1{\cdot}5$ on sketch • time estimated using relevant values other than those required for the maximum time. **Notes:** (i) *candidates may show relevant work for this section on graph* (ii) *accept values from candidate's graph*