Question 9 **(50 marks)**

Data on earnings were published for a particular country. The data showed that the annual income of people in full-time employment was normally distributed with a mean of €39 400 and a standard deviation of €12 920.

(a) **(i)** The government intends to impose a new tax on incomes over €60 000. Find the percentage of full-time workers who will be liable for this tax, correct to one decimal place.

(ii) The government will also provide a subsidy to the lowest 10 % of income earners. Find the level of income at which the government will stop paying the subsidy, correct to the nearest euro.

(iii) Some time later a research institute surveyed a sample of 1000 full-time workers, randomly selected, and found that the mean annual income of the sample was €38 280. Test the hypothesis, at the 5 % level of significance, that the mean annual income of full-time workers has changed since the national data were published.
State the null hypothesis and the alternative hypothesis.
Give your conclusion in the context of the question.

(b) The research institute surveyed 400 full-time farmers, randomly selected from all the full-time farmers in the country, and found that the mean income for the sample was €26 974 and the standard deviation was €5120.
Assuming that annual farm income is normally distributed in this country, create a 95 % confidence interval for the mean income of full-time farmers.

(c) It is known that data on farm size are not normally distributed.
The research institute could take many large random samples of farm size and create a sampling distribution of the means of all these samples.
Give one reason why they might do this.

(d) The research institute also carried out a survey into the use of agricultural land.
n farmers were surveyed.
If the margin of error of the survey was 4·5 %, find the value of n.

Q9	Model Solution – 50 Marks	Marking Notes
(a) **(i)**	$\mu = 39400, \;\; \sigma = 12920$ $z = \dfrac{x-\mu}{\sigma} = \dfrac{60000 - 39400}{12920}$ $z = 1{\cdot}59$ $P(z > 1{\cdot}59) = 1 - P(z < 1{\cdot}59)$ $= 1 - 0{\cdot}9441 = 0{\cdot}0559$ $= 5{\cdot}59\%$ = 5·6%	Scale 10D (0, 3, 5, 8, 10) *Low Partial Credit* • μ and σ identified *Mid Partial Credit* • z = 1·59 *High Partial Credit* • identifies 0·9441
(a) **(ii)**	$P(z \leq z_1) = 0{\cdot}9$ $z_1 = 1{\cdot}28$ $\Longrightarrow z_2 = -1{\cdot}28$ $\Longrightarrow \dfrac{x - 39400}{12920} = -1{\cdot}28$ $x = 22862{\cdot}40$ = €22 862	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • identifies 1·28 but fails to progress *High Partial Credit* • formula for x fully substituted
(a) **(iii)**	$\mu = 39400, \;\; \sigma = 12920,$ $\bar{x} = 38280, \;\; n = 1000$ $H_0 \Rightarrow \mu = 39400$ $H_1 \Rightarrow \mu \neq 39400$ $z = \dfrac{38280 - 39400}{\dfrac{12920}{\sqrt{1000}}} = -2{\cdot}74$ $-2{\cdot}74 < \; -1{\cdot}96$ Result is significant. There is evidence to reject the null hypothesis The mean income has changed.	Scale 15D (0, 4, 7, 11,15) *Low Partial Credit* • z formulated with some substitution • states null and/or alternative hypothesis only • reference to 1·96 Mid Partial Credit • z fully substituted *High Partial Credit* • $z = -2{\cdot}74$ and stops • fails to state the null and alternative hypothesis correctly • fails to contextualise the answer

	or Confidence Interval: $$\bar{x} \pm 1{\cdot}96\frac{\sigma}{\sqrt{n}}$$ $$39400 \pm 1{\cdot}96\frac{12920}{\sqrt{1000}}$$ $$[38599{\cdot}2,\ 40200{\cdot}8]$$ 38280 outside range Result is significant. There is evidence to reject the null hypothesis The mean income has changed. **or** Confidence Interval: $$\bar{x} \pm 1{\cdot}96\frac{\sigma}{\sqrt{n}}$$ $$38280 \pm 1{\cdot}96\frac{12920}{\sqrt{1000}}$$ $$38280 \pm 1{\cdot}96(408{\cdot}57)$$ $$[37479{\cdot}2,\ 39080{\cdot}8]$$ 39400 outside range Result is significant. There is evidence to reject the null hypothesis The mean income has changed.	

Q9		Marking Notes
(b)	$26974 - 1{\cdot}96\left(\frac{5120}{\sqrt{400}}\right) \leq \mu \leq 26974 + 1{\cdot}96\left(\frac{5120}{\sqrt{400}}\right)$ $26472{\cdot}24 \leq \mu \leq 27475{\cdot}76$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • interval formulated with some correct substitution *High Partial Credit* • interval formulated with fully correct substitution
(c)	The distribution of sample means will be normally distributed	Scale 5B (0, 2, 5) *Partial Credit* • mentions 30 (or more) but not contextualised
(d)	$\frac{1}{\sqrt{n}} = 0{\cdot}045$ $\frac{1}{0{\cdot}045} = \sqrt{n}$ $n = \left(\frac{1}{0{\cdot}045}\right)^2 = 493{\cdot}827$	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • $\frac{1}{\sqrt{n}}$ *High Partial Credit* • n formulated with fully correct substitution **Note:** Accept 493 farmers or 494 farmers