Question 2 **(25 marks)**

$z = -\sqrt{3} + i$, where $i^2 = -1$.

(a) Use De Moivre's Theorem to write z^4 in the form $a + b\sqrt{c}\, i$, where a, b, and $c \in \mathbb{Z}$.

(b) The complex number w is such that $|w| = 3$ and w makes an angle of $30°$ with the positive sense of the real axis. If $t = zw$, write t in its simplest form.

Q2	Model Solution – 25 Marks	Marking Notes
(a)	$z = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$ $z^4 = \left(2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)\right)^4$ $z^4 = 16\left(\cos\frac{10\pi}{3} + i\sin\frac{10\pi}{3}\right)$ $= -8 - 8\sqrt{3}i$	**Scale 15D (0, 5, 8, 12, 15)** *Low Partial Credit:* • θ or $\lvert z\rvert$ found *Mid Partial Credit:* • z written in polar form *High Partial Credit:* • De Moivre's Theorem applied correctly **Note***:* Not using De Moivre: Low partial credit for fully correct work
(b)	$w = 3(\cos 30 + i\sin 30)$ $zw = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) \times 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ $zw = 6(\cos\pi + i\sin\pi)$ $= 6(-1 + 0i)$ $= -6$ **OR (contd)**	**Scale 10D (0, 4, 7, 8, 10)** *Low Partial Credit:* • Work towards w in Cartesian or polar form *Mid Partial Credit* • zw expressed as a product *High Partial Credit:* • zw in Cartesian or polar form

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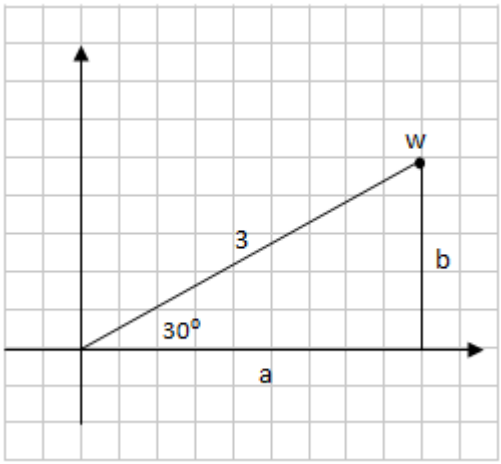

$$w = a + bi$$

$$a^2 + b^2 = 9$$

$$\frac{b}{3} = \sin 30° = \frac{1}{2}$$

$$b = \frac{3}{2}$$

$$a^2 + \left(\frac{3}{2}\right)^2 = 9$$

$$a^2 = \frac{27}{4}$$

$$a = \sqrt{\frac{27}{4}} = \frac{3\sqrt{3}}{2}$$

$$w = a + bi = \frac{3\sqrt{3}}{2} + \frac{3}{2}i$$

$$z = -\sqrt{3} + i$$

$$zw = \left(-\sqrt{3} + i\right)\left(\frac{3\sqrt{3}}{2} + \frac{3}{2}i\right)$$

$$= -\frac{9}{2} - \frac{3\sqrt{3}i}{2} + \frac{3\sqrt{3}i}{2} - \frac{3}{2}$$

$$= -6$$

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