Question 3 **(25 marks)**

(a) Differentiate $\frac{1}{3}x^2 - x + 3$ from first principles with respect to x.

(b) $f(x) = \ln(3x^2 + 2)$ and $g(x) = x + 5$, where $x \in \mathbb{R}$.
Find the value of the derivative of $f\big(g(x)\big)$ at $x = \frac{1}{4}$.
Give your answer correct to 3 decimal places.

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<tr><th>Q3</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>

$$f(x+h) = \frac{1}{3}(x+h)^2 - (x+h) + 3$$

$$f(x) = \frac{1}{3}x^2 - x + 3$$

$$f(x+h) - f(x) = \frac{2xh}{3} + \frac{h^2}{3} - h$$

$$\frac{f(x+h) - f(x)}{h} = \frac{2x}{3} + \frac{h}{3} - 1$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{2x}{3} - 1$$

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Scale 20D (0, 5, 14, 17, 20)

Low Partial Credit

- any $f(x+h)$

Mid Partial Credit

- $f(x+h) - f(x)$ *with some correct work*

High Partial Credit

- $\frac{\frac{1}{3}(x+h)^2 - (x+h) + 3 - \left(\frac{x^2}{3} - x + 3\right)}{h}$ simplified

Notes*:*

- omission of limit sign penalised once only
- answer not from 1st Principles merits 0 marks

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<tr><td>(b)</td><td>

$$\frac{d(fg(x))}{dx} =$$

$$\frac{1}{(3(x+5)^2 + 2)}(6(x+5))$$

$$\frac{d(fg\left(\frac{1}{4}\right))}{dx} = \frac{6(\frac{21}{4})}{3(\frac{21}{4})^2 + 2} = \frac{504}{1355}$$

$$= 0{\cdot}372$$

OR

$$f(x) = \ln(3x^2 + 2)$$

$$g(x) = (x+5)$$

$$f[g(x)] = \ln[3(x+5)^2 + 2]$$

$$= \ln(3x^2 + 30x + 77)$$

$$f'(x) = \frac{6x + 30}{3x^2 + 30x + 77}$$

$$x = \frac{1}{4}: \quad f'(x) = \frac{31{\cdot}5}{84{\cdot}6875} = 0{\cdot}3719$$

$$= 0{\cdot}372$$

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Scale 5C (0, 3, 4, 5)

Low Partial Credit:

- Any correct differentiation
- $fg(x)$ formulated

High Partial Credit:

- $\frac{d(fg(x)}{dx}$ found

Note*:*

Work with $f(x) \times g(x)$ merits low partial credit at most

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