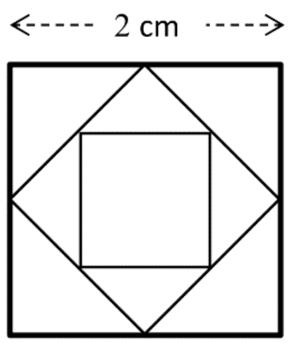
Question 4 **(25 marks)**

(a) The amount of a substance remaining in a solution reduces exponentially over time.
An experiment measures the percentage of the substance remaining in the solution.
The percentage is measured at the same time each day. The data collected over the first 4 days are given in the table below. Based on the data in the table, estimate which is the first day on which the percentage of the substance in the solution will be less than 0·01%.

Day	1	2	3	4
Percentage of substance (%)	95	42·75	19·2375	8·6569

(b) A square has sides of length 2 cm. The midpoints of the sides of this square are joined to form another square. This process is continued.
The first three squares in the process are shown below.
Find the sum of the perimeters of the squares if this process is continued indefinitely.
Give your answer in the form $a + b\sqrt{c}$ cm, where a, b, and $c \in \mathbb{N}$.



Q4	Model Solution – 25 Marks	Marking Notes
(a)	$r = \frac{42{\cdot}75}{95} = \frac{9}{20}$ $\quad T_n = ar^{n-1} < 0{\cdot}01$ $95\left(\frac{9}{20}\right)^{n-1} < 0{\cdot}01$ $\left(\frac{9}{20}\right)^{n-1} < \frac{0{\cdot}01}{95}$ $(n-1)\log\left(\frac{9}{20}\right) < \log\left(\frac{0.01}{95}\right)$ $(n-1) > \frac{\log\left(\frac{0.01}{95}\right)}{\log\left(\frac{9}{20}\right)}$ (since $\log\left(\frac{9}{20}\right)$ is negative) $n-1 > 11{\cdot}47$ $n > 12{\cdot}47$ 12th day	**Scale 15D (0, 5, 8, 12, 15)** *Low Partial Credit:* • r found • T_n of a GP with some substitution *Mid Partial Credit:* • Inequality in n written *High Partial Credit:* • Inequality in n simplified (log handled) *Full Credit:* • Accept $n = 12{\cdot}47$
(b)	$4(2) + 4\sqrt{2} + 4 + \cdots\cdots\cdots$ $a = 8 \quad r = \frac{1}{\sqrt{2}}$ $S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{8}{1-\frac{1}{\sqrt{2}}}$ $S_\infty = \frac{8}{1-\frac{1}{\sqrt{2}}} \cdot \frac{1+\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$ $S_\infty = \frac{8\left(1+\frac{1}{\sqrt{2}}\right)}{\frac{1}{2}}$ $S_\infty = 16 + 8\sqrt{2}$	**Scale 10C (0, 5, 8, 10)** *Low Partial Credit:* • length of one side of new square *High Partial Credit:* • S_∞ fully substituted • Correct work with one side only