Question 5 **(25 marks)**

The function f is such that $f(x) = 2x^3 + 5x^2 - 4x - 3$, where $x \in \mathbb{R}$.

(a) Show that $x = -3$ is a root of $f(x)$ **and** find the other two roots.

(b) Find the co-ordinates of the local maximum point **and** the local minimum point of the function f.

(c) $f(x) + a$, where a is a constant, has only one real root.
Find the range of possible values of a.

Q5	Model Solution – 25 Marks	Marking Notes
(a)	$f(x) = 2x^3 + 5x^2 - 4x - 3$ $f(-3) = 2(-3)^3 + 5(-3)^2 - 4(-3) - 3$ $= -54 + 45 + 12 - 3$ $f(-3) = 0$ $\Rightarrow (x+3)$ is a factor $$\begin{array}{r} 2x^2 - x - 1 \\ x+3 \overline{)\, 2x^3 + 5x^2 - 4x - 3} \\ \underline{2x^3 + 6x^2} \qquad\qquad \\ -x^2 - 4x \qquad \\ \underline{-x^2 - 3x} \qquad \\ -x - 3 \\ \underline{-x - 3} \end{array}$$ $f(x) = (x+3)(2x^2 - x - 1)$ $f(x) = (x+3)(2x+1)(x-1)$ $x = -3 \quad x = -\frac{1}{2} \quad x = 1$	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* • Shows $f(-3) = 0$ *High Partial Credit:* • quadratic factor of $f(x)$ found Note: No remainder in division may be stated as reason for $x = -3$ as root

(b)	$$y = 2x^3 + 5x^2 - 4x - 3$$ $$\frac{dy}{dx} = 6x^2 + 10x - 4 = 0$$ $$3x^2 + 5x - 2 = 0$$ $$(x+2)(3x-1) = 0$$ $$3x - 1 = 0 \quad x + 2 = 0$$ $$x = \frac{1}{3} \quad x = -2$$ $$f\left(\frac{1}{3}\right) = \frac{-100}{27} \quad f(-2) = 9$$ $$Max = (-2, 9) \quad Min = \left(\frac{1}{3}, \frac{-100}{27}\right)$$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • $\frac{dy}{dx}$ found (Some correct differentiation) *High Partial Credit* • roots and one y value found Note: One of Max/Min must be identified for full credit
(c)	$$a > \frac{100}{27} \ \textbf{ or } \ a < -9$$	**Scale 5B (0, 3, 5)** *Partial Credit:* • one value identified • no range identified (from 2 values)