Question 8 **(55 marks)**

(a) When a loan of €P is repaid in equal repayments of amount €A, at the end of each of t equal periods of time, where i is the periodic compound interest rate (expressed as a decimal), the formula below can be used to find the amount of each repayment.

$$A = P\frac{i(1+i)^t}{((1+i)^t - 1)}$$

Show how this formula is derived. You may use the formula for the sum of a finite geometric series.

(b) Alex has a credit card debt of €5000. One method of clearing this debt is to make a fixed repayment at the end of each month. The amount of this repayment is 2·5% of the original debt.

(i) What is the fixed monthly repayment, €A, required to pay the debt of €5000?

(ii) The annual percentage rate (APR) charged on debt by the credit card company is 21·75%, fixed for the term of the debt. Find as a percentage, correct to 3 significant figures, the monthly interest rate that is equivalent to an APR of 21·75%.

(iii) Assume Alex pays the fixed monthly repayment, €A, each month and does not have any further transactions on that card. Complete the table below to show how the balance of the debt of €5000 is reducing each month for the first three months, assuming an APR of 21·75%, charged and compounded monthly.

Payment number	Fixed monthly payment, €A	€A		New balance of debt (€)
		Interest	Previous balance reduced by (€)	
0				5000
1			42·50	4957·50
2				
3				

(iv) Using the formula you derived on the previous page, or otherwise, find how long it would take to pay off a credit card debt of €5000, using the repayment method outlined at the beginning of **part (b)** above.
Give your answer in months, correct to the nearest month.

(v) Alex decides to borrow €5000 from the local Credit Union to pay off this credit card debt of €5000. The APR charge for the Credit Union loan is 8·5% fixed for the term of the loan. The loan is to be repaid in equal weekly repayments, at the end of each week, for 156 weeks. Find the amount of each weekly repayment.

(vi) How much will Alex save by paying off the credit card debt using the loan from the Credit Union instead of paying the fixed repayment from **part (b)(i)** each month to the credit card company?

Q8	Model Solution – 55 Marks	Marking Notes
(a)	$$P=\frac{A}{1+i}+\frac{A}{(1+i)^2}+\cdots\cdots+\frac{A}{(1+i)^t}$$ $$P=\frac{\left(\frac{A}{1+i}\right)\left(1-\left(\frac{1}{1+i}\right)^t\right)}{1-\frac{1}{1+i}}$$ $$=\frac{A\left(1-\frac{1}{(1+i)^t}\right)}{1+i-1}$$ $$=\frac{A((1+i)^t-1)}{i(1+i)^t}$$ $$A=\frac{P(i)(1+i)^t}{(1+i)^t-1}$$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • $P=\frac{A}{1+i}$ • $A=P(1+i)$ • S_n formula with some substitution *High Partial Credit:* • full substitution for P (or A) into S_n formula.
(b) (i)	$2{\cdot}5\% \times 5000 = 125$	**Scale 10B (0, 4, 10)** *Partial Credit* • Any one unknown
(b) (ii)	$(1+i)^{\frac{1}{12}}=(1{\cdot}2175)^{\frac{1}{12}}=1{\cdot}016535$ $Rate = 1{\cdot}65\%$	**Scale 10B (0, 4, 10)** *Partial Credit* • Formula with some substitution

(b) (iii)

Payment number	Fixed monthly payment, €A	€A		New balance of debt (€)
		Interest	Previous balance reduced by (€)	
0				5000
1	125	82·50	42·50	4957·50
2	125	81·80	43·20	4914·30
3	125	81·09	43·91	4870·39

(b) (iii)		**Scale 10C (0, 5, 8, 10)** *Low Partial Credit:* • One correct additional entry *High Partial Credit:* • 6 correct additional entries **Note:** Where interest rate in b(ii) is not 1·65%, then check the validity of all values given.

(iv)		
(iv)	$$A = p\left[\frac{i(1+i)^t}{(1+i)^t - 1}\right]$$ $$A[(1+i)^t - 1] = pi(1+i)^t$$ $$A(1+i)^t - A = pi(1+i)^t$$ $$A = (1+i)^t[A - pi]$$ $$\frac{A}{A-pi} = (1+i)^t$$ $$\frac{125}{125 - 5000\left(\frac{1{\cdot}65}{100}\right)} = \left(1 + \frac{1{\cdot}65}{100}\right)^t$$ $$\frac{125}{42{\cdot}5} = (1{\cdot}0165)^t$$ $$\log\left(\frac{125}{42{\cdot}5}\right) = t\log(1{\cdot}0165)$$ $$t = \frac{\log\left(\frac{125}{42{\cdot}5}\right)}{\log(1{\cdot}0165)}$$ $$t = 65{\cdot}920$$ t = 66 months **OR** $$A = p\left[\frac{i(1+i)^t}{(1+i)^t - 1}\right]$$ $$125 = \frac{5000(0{\cdot}0165)(1{\cdot}0165)^t}{(1{\cdot}0165)^t - 1}$$ $$125 = \frac{82{\cdot}5(1{\cdot}0165)^t}{(1{\cdot}0165)^t - 1}$$ $$\frac{125}{82{\cdot}5} = \frac{1{\cdot}0165^t}{1{\cdot}0165^t - 1}$$ $$\frac{50}{33} = \frac{1{\cdot}0165^t}{1{\cdot}0165^t - 1}$$ $$50(1\cdot0165^t - 1) = 33(1{\cdot}0165^t)$$ $$50(1{\cdot}0165^t) - 50 = 33(1{\cdot}0165^t)$$ $$50(1{\cdot}0165^t) - 33(1{\cdot}0165^t) = 50$$ $$1{\cdot}0165^t(50 - 33) = 50$$ $$1{\cdot}0165^t(17) = 50$$ $$1{\cdot}0165^t = \frac{50}{17}$$ $$t\log 1{\cdot}0165 = \log\frac{50}{17}$$ $$t = \frac{\log\left(\frac{50}{17}\right)}{\log 1{\cdot}0165} = 65{\cdot}92$$ t = 66 months	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • Formula with some substitution • Some relevant manipulation of formula. *High Partial Credit:* • Equation in t (t no longer an index)

(v)	$$A = \frac{pi(1+i)^t}{(1+i)^t - 1}$$ $$= \frac{5000\left(1{\cdot}085^{\frac{1}{52}} - 1\right)(1{\cdot}085)^3}{(1{\cdot}085)^3 - 1}$$ $$= €36{\cdot}16$$ **OR** Weekly interest rate $(1+i)^{52} = 1{\cdot}085$ $$1 + i = 1{\cdot}085^{\frac{1}{52}}$$ $$1 + i = 1{\cdot}00157$$ $$i = 0{\cdot}00157$$ $$A = \frac{pi(1+i)^t}{(1+i)^t - 1}$$ $$A = \frac{5000(0{\cdot}00157)(1{\cdot}00157)^{156}}{(1{\cdot}00157)^{156} - 1}$$ $$= €36{\cdot}16$$	**Scale 10C (0, 5, 8, 10)** *Low Partial Credit:* • r (weekly) found *High Partial Credit:* • Fully substituted equation
(vi)	$$125 \times 66 - (36{\cdot}16)(156)$$ =€2609·04	**Scale 5B (0, 3, 5)** *Partial Credit:* • Total repayment by either method found