Question 9 **(40 marks)**

The depth of water, in metres, at a certain point in a harbour varies with the tide and can be modelled by a function of the form

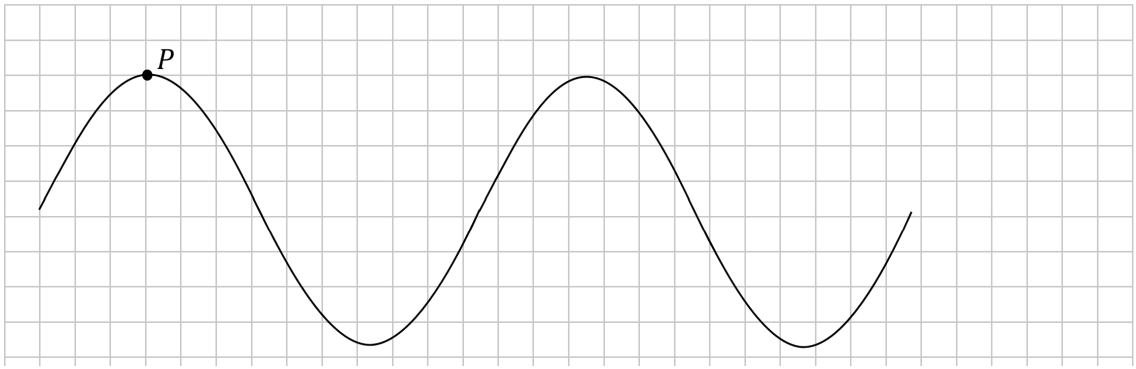
$$f(t) = a + b\cos ct$$

where t is the time in hours from the first high tide on a particular Saturday and a, b, and c are constants. (**Note**: ct is expressed in radians.)

On that Saturday, the following were noted:

- The depth of the water in the harbour at high tide was 5·5 m
- The depth of the water in the harbour at low tide was 1·7 m
- High tide occurred at 02:00 and again at 14:34.

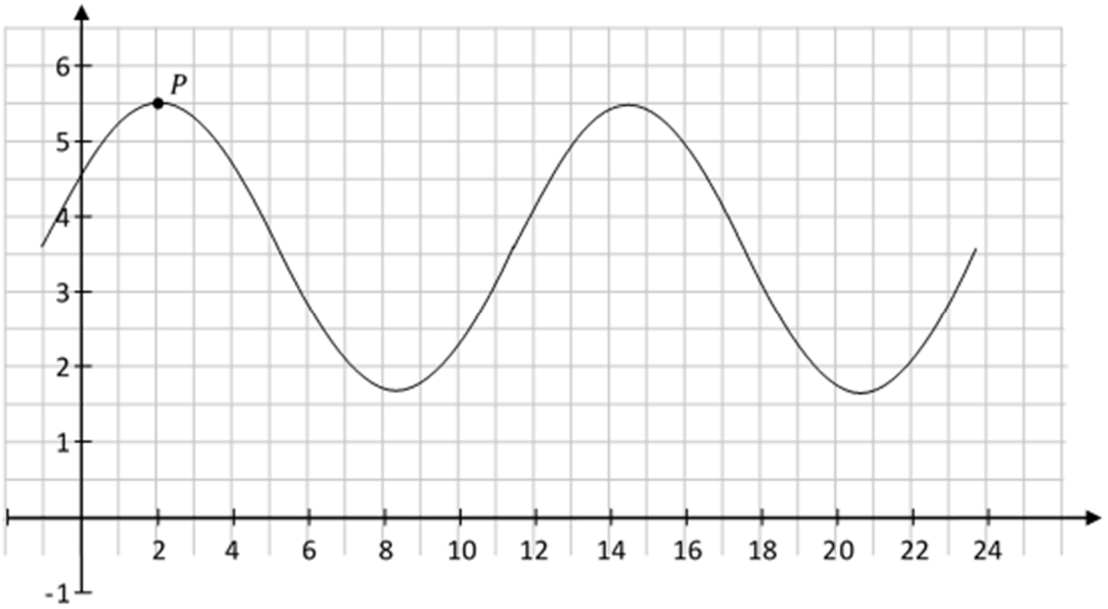
(a) Use the information you are given to add, as accurately as you can, labelled and scaled axes to the diagram below to show the graph of f over a portion of that Saturday.
The point P should represent the depth of the water in the harbour at high tide on that Saturday morning.



(b) **(i)** Find the value of a and the value of b.

(ii) Show that $c = 0{\cdot}5$, correct to 1 decimal place.

(c) Use the equation $f(t) = a + b\cos ct$ to find the times on that Saturday **afternoon** when the depth of the water in the harbour was exactly 5·2 m.
Give each answer correct to the nearest minute.

Q9	Model Solution – 40 Marks	Marking Notes
(a)		
(a)		**Scale 20C (0, 10, 18, 20)** *Low Partial Credit:* • Vertical axis drawn • Horizontal axis drawn. *High Partial Credit:* • Horizontal axis fully scaled and positioned **OR** • Vertical axis fully scaled Use relevant portions of axes **Note:** *P* can be on vertical axis

[20]

Q9		Marking Notes
(b) (i)	$f(t) = a + b\cos ct$ $Range$: $[(a+b),(a-b)]$ $a + b = 5{\cdot}5 \quad a - b = 1{\cdot}7$ $a = 3{\cdot}6 \quad b = 1{\cdot}9$	**Scale 10C (0, 5, 8, 10)** *Low Partial Credit:* • one equation in a and b • Range in terms of a and b *High Partial Credit:* • a or b found Note: Accept correct answer without work
(b) (ii)	Time between two successive high tides is: $12\frac{34}{60}$ hours $\text{period} = 12\frac{34}{60}$ $\text{period} = \frac{2\pi}{c}$ $c = \frac{2\pi}{12\frac{34}{60}} = 0{\cdot}4999 = 0{\cdot}5$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • Period identified **or** $\frac{2\pi}{c}$ **or** $12{\cdot}34$ *High Partial Credit:* • equation in c with some substitution
(c)	$5{\cdot}2 = a + b\cos ct$ $5{\cdot}2 = 3{\cdot}6 + 1{\cdot}9\cos 0{\cdot}5t$ $0{\cdot}5t = \cos^{-1}\frac{1{\cdot}6}{1{\cdot}9} = 0{\cdot}569621319$ $0{\cdot}5t = 0{\cdot}5696$ $t = 1{\cdot}139$ hours (before and after high tide at 14:34) Time = 1 hour 8 minutes Times: (14:34) ± 1 hour 8 min ⇒ 13:26 and 15:42	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • equation with some substitution *High Partial Credit:* • solution for t **Note***:* Low partial at most if formula not used