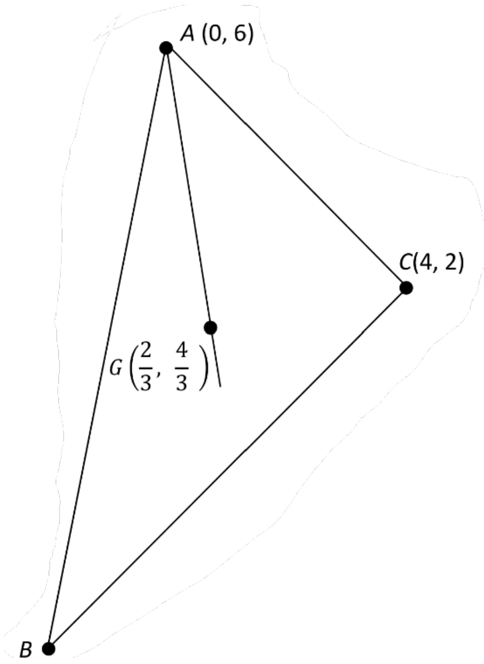
Question 3

ABC is a triangle where the co-ordinates of *A* and *C* are (0, 6) and (4, 2) respectively.
$G\left(\frac{2}{3}, \frac{4}{3}\right)$ is the centroid of the triangle *ABC*.
AG intersects *BC* at the point *P*.
$|AG| : |GP| = 2 : 1$.



(a) Find the co-ordinates of *P*.

(b) Find the co-ordinates of *B*.

(c) Prove that *C* is the orthocentre of the triangle *ABC*.

Q3	Model Solution – 25 Marks	Marking Notes
(a)	$A(0,6) \to G\left(\frac{2}{3}, \frac{4}{3}\right)$ $\to P\left(\frac{2}{3}+\frac{1}{2}\left(\frac{2}{3}\right), \frac{4}{3}+\frac{1}{2}\left(\frac{-14}{3}\right)\right)$ $= \left(\frac{3}{3}, -\frac{3}{3}\right)$ $P = (1, -1)$ **or** $P = (x, y)$ $\left(\frac{2x+1(0)}{3}, \frac{2y+6}{3}\right) = \left(\frac{2}{3}, \frac{4}{3}\right)$ $x = 1, \quad y = -1$ **or** $P = (x, y)$ $\left(\frac{3(\frac{2}{3})-1(0)}{3-1}, \frac{3\left(\frac{4}{3}\right)-1(6)}{3-1}\right)$ $= \left(\frac{2}{2}, \frac{-2}{2}\right) = (1, -1)$	**Scale 10C (0, 4, 5, 10)** *Low Partial Credit:* • $P\left(\frac{4}{3}, -\frac{10}{3}\right)$ or equivalent, i.e ratio 1:1 • $\frac{2}{3}$ or $\frac{1}{3}$ identified as part of change in x ordinate • $-\frac{14}{3}$ or $-\frac{7}{3}$ identified as part of change in y ordinate • Ratio formula with some substitution *High Partial Credit:* • one relevant co-ordinate of P found
(b)	$C(4,2) \to P(1, -1) \to B(1-3, -1-3)$ $= (-2, -4)$ $B(x,y) \to \left(\frac{4+x}{2}, \frac{2+y}{2}\right) = (1, -1)$ $x = -2, \quad y = -4$ $B = (-2, -4)$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* • P as mid-point of BC *High Partial Credit:* • one relevant co-ordinate of B found ***Note:*** Accept (–2, –4) without work Accept correct graphical solution

(c)	$AC \perp BC$ $AC = \frac{2-6}{4-0} = -1$ $BC = \frac{2+4}{4+2} = 1$ $-1 \times 1 = -1$ lines are perpendicular **or** $\text{Slope AB} = 5.$ $\text{Altitude from C}: y - 2 = -\frac{1}{5}(x - 4)$ $\rightarrow x + 5y = 14 \ldots. \text{(i)}.$ $\text{Slope AC} = -1.$ $\text{Altitude from B}:$ $y + 4 = 1(x + 2)$ $\rightarrow x - y = 2 \ldots \ldots \text{(ii)}$ $\rightarrow \text{Solving (i)and (ii)}$ $x = 4$ $y = 2$	**Scale 10C (0, 4, 5, 10)** *Low Partial Credit:* • Identifies significance of right-angled triangle • one equation of perpendicular from vertex to opposite side found *High Partial Credit:* • slope of AC and slope of BC found but no conclusion • two equations of perpendiculars from vertex to opposite side found