Question 4 **(25 marks)**

$A(0, 0)$, $B(6{\cdot}5, 0)$ and $C(10, 7)$ are three points on a circle.

(a) Find the equation of the circle.

(b) Find $|\angle BCA|$. Give your answer in degrees, correct to 2 decimal places.

Q4	Model Solution – 25 Marks	Marking Notes
(a)	$x^2+y^2+2gx+2fy+c=0$ $(0,0)\Rightarrow 0+0+0+0+c=0$ $\Rightarrow\ c=0$ $(6\cdot5,0)\Rightarrow 42\cdot25+0+13g+0+0=0$ $\Rightarrow g=-3\cdot25$ $(10,7)\Rightarrow 100+49+2(-3\cdot25)(10)$ $+2f(7)=0$ $14f=-84$ $f=-6$ $x^2+y^2-6\cdot5x-12y=0$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • $c=0$ • One relevant equation in *g* and/or *f* *Mid Partial Credit:* • 2 of *g, f, c* found *High Partial Credit:* • *g , f*, and *c* found or equivalent
	or $\perp$ Bisector of [*AB*] $x=\frac{13}{4}$ (l_1) $\perp$ Bisector of [*AC*] Midpoint [*AC*] $=\left(5,\ \frac{7}{2}\right)$, Slope [*AC*] $=\frac{7}{10}$ Eq. of mediator [*AC*] $y-\frac{7}{2}=-\frac{10}{7}(x-5)$ $10x+7y=\frac{149}{2}$ (l_2) $l_1\cap l_2=\left(\frac{13}{4},6\right)$ $r=\sqrt{\left(\frac{13}{4}-0\right)^2+(6-0)^2}=\frac{\sqrt{745}}{4}$ $\left(x-\frac{13}{4}\right)^2+(y-6)^2=\frac{745}{16}$	*Low Partial Credit:* • Effort at formulating equation of 1 $\perp$ bisector *Mid Partial Credit:* • Point t of intersection of 2 $\perp$ bisectors found *High Partial Credit:* • Point of intersection of 2 $\perp$ bisectors and radius
	or $(-g,-f)\in$ mediator $(0,0)$and $(6\cdot5,0)$. $\therefore -g=3\cdot25$ Centre $(3\cdot25,-f)$. Since $(0,0)\in$ of circle $\therefore c=0$. Equation of circle $x^2+y^2-6\cdot5x+2fy+0=0$ $(10,7)$on circle: $100+49-65+14f=0$ $84+14f=0$ $f=-6$ $x^2+y^2-6\cdot5x-12y=0$	*Low Partial Credit:* • $c=0$ • One point substituted into equation of circle • Midpoint (0,0) and $(6\cdot5,0)$ formulated *Mid Partial Credit:* • 2 of *g, f, c* found *High Partial Credit:* • *g , f*, and c found or equivalent

(b)	Slope $AC = \frac{7}{10}$ Slope $CB = \frac{0-7}{6\cdot5-10} = 2$ $$\tan\theta = \pm\frac{\frac{7}{10}-2}{1+\frac{7}{5}} = \pm\frac{-13}{24}$$ $$\theta = 28\cdot44$$ **or** Cosine rule $\lvert AB\rvert^2 = 42\cdot25,$ $\lvert AC\rvert^2 = 149$ $\lvert BC\rvert^2 = 61\cdot25$ $$\cos\theta = \frac{149+61\cdot25-42\cdot25}{2\times\sqrt{149}\times\sqrt{61\cdot25}} = 0\cdot8793$$ $$\Rightarrow \theta = 28\cdot44$$	**Scale 15C (0, 6, 9, 15)** *Low Partial Credit:* • one relevant slope *High Partial Credit:* • $\tan\theta$ fully substituted *Low Partial Credit:* • one relevant length *High Partial Credit:* • $\cos\theta$ fully substituted