Question 7 **(40 marks)**

Two solid cones, each of radius R cm and height R cm are welded together at their vertices and placed in the smallest possible hollow cylinder, as shown in **Figure 1** below.

Figure 1

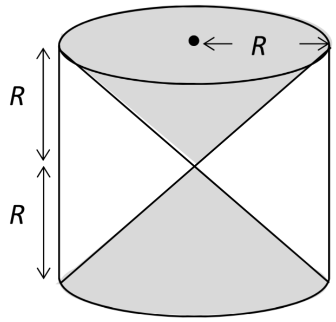
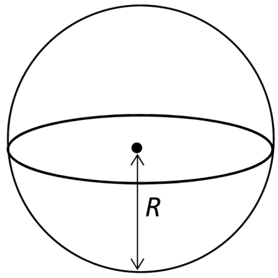


Figure 2



(a) Show that the capacity (volume) of the empty space in the cylinder is equal to the capacity of an empty sphere of radius R cm (**Figure 2**).

(b) In the remainder of this question, $R = 12$ cm. Water is poured into both the cylinder and the sphere to a depth of 6 cm as shown below (**Figure 3** and **Figure 4** respectively).

(i) Find $|AB|$, the radius of the circular surface of the water in the sphere (**Figure 4**). Give your answer in the form $a\sqrt{b}$ cm, where $a, b \in \mathbb{N}$.

Figure 3

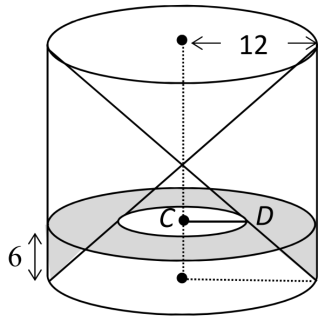
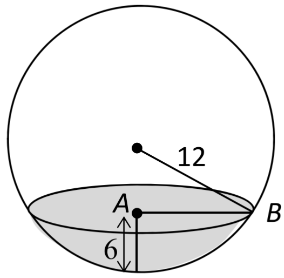


Figure 4



(ii) Find $|CD|$, the radius of the cone at water level, as shown in **Figure 3**.

(iii) Verify that the area of the surface of the water in the sphere is equal to the area of the surface of the water in the cylinder.

(c) The mathematician Cavalieri discovered that, at the same depth, the volume of water in the available space in the cylinder is equal to the volume of water in the sphere.
Use this discovery to find the volume of water in the sphere when the depth is 6 cm.
Give your answer in terms of π.

Q7	**Model Solution – 40 Marks**	**Marking Notes**
(a)	$\text{Vol of space} = \text{Cylinder} - 2\times\text{Cone}$ $= \pi R^2(2R) - \frac{2}{3}\pi R^2(R)$ $= 2\pi R^3 - \frac{2}{3}\pi R^3$ $= \frac{4}{3}\pi R^3$	**Scale 10C (0, 4, 5, 10)** *Low Partial Credit:* • A relevant volume formulated *High Partial Credit:* • Vol of space formulated in terms of π and R
(b) (i)	$12^2 = 6^2 + \lvert AB\rvert^2$ $\lvert AB\rvert = \sqrt{12^2 - 6^2} = \sqrt{108} = 6\sqrt{3}$	**Scale 10B (0, 5, 10)** *Partial Credit:* • indication of Pythagoras
(b) (ii)	$\frac{h_1}{h_2} = \frac{6}{12} = \frac{r}{12}$ $r = 6 \text{ cm}$	**Scale 10C (0, 4, 5, 10)** *Low Partial Credit:* • indication of similar triangles • indication of a relevant ratio *High Partial Credit:* • corresponding ratios identified but fails to finish **Note**: Accept correct answer without work
(b) (iii)	$\text{Cylinder} = \pi 12^2 - \pi 6^2 = 108\pi$ $\text{Sphere} = \pi(6\sqrt{3})^2 = 108\pi$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* • Surface Area in Fig. 3 substituted • Surface Area in Fig 4 substituted *High Partial Credit:* • One Surface Area found
(c)	$\text{Vol} = \pi(12^2)(6)$ $-\left(\frac{1}{3}\pi 12^2 \times 12 - \frac{1}{3}\pi 6^2 \times 6\right)$ $\text{Vol} = 360\pi \text{ cm}^3$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* • Vol of cylinder found • Vol of truncated cone substituted • Vol of one cone found (12 or 6) *High Partial Credit:* • Volume fully substituted but fails to finish • Volume of truncated cone found