Question 8 **(60 marks)**

(a) In 2015, in a particular country, the weights of 15 year olds were normally distributed with a mean of 63·5 kg and a standard deviation of 10 kg.

(i) In 2015, Mariska was a 15 year old in that country. Her weight was 50 kg.
Find the percentage of 15 year olds in that country who weighed more than Mariska.

(ii) In 2015, Kamal was a 15 year old in that country.
1·5% of 15 year olds in that country were heavier than Kamal.
Find Kamal's weight.

(iii) In 2016, 150 of the 15 year olds in that country were randomly selected and their weights recorded. It was found that their weights were normally distributed with a mean weight of 62 kg and a standard deviation of 10 kg. Test the hypothesis, at the 5% level of significance, that the mean weight of 15 year olds, in that country, had not changed from 2015 to 2016. State the null hypothesis and your alternative hypothesis. Give your conclusion in the context of the question.

(b) In Galway, rain falls in the morning on $\frac{1}{3}$ of the school days in the year.

When it is raining the probability of heavy traffic is $\frac{1}{2}$.

When it is not raining the probability of heavy traffic is $\frac{1}{4}$.

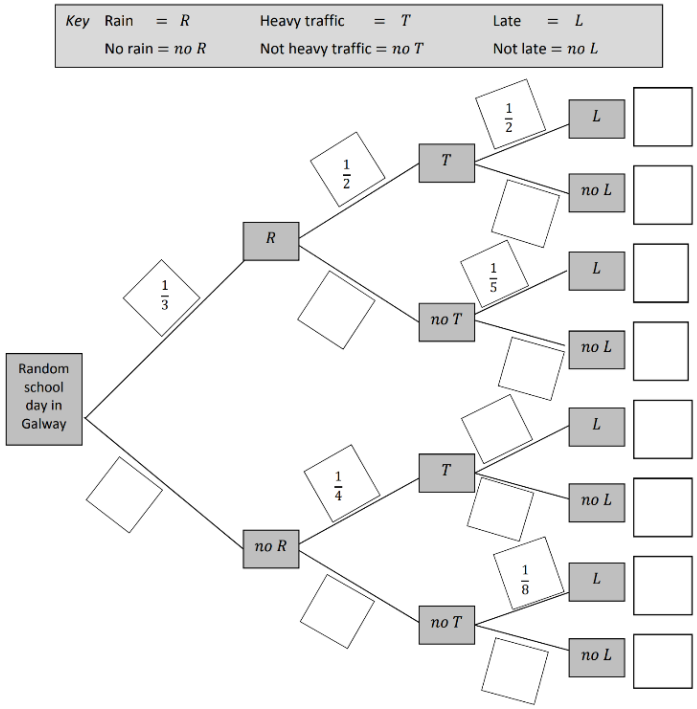
When it is raining and there is heavy traffic, the probability of being late for school is $\frac{1}{2}$.

When it is not raining and there is no heavy traffic, the probability of being late for school is $\frac{1}{8}$.

In any other situation the probability of being late for school is $\frac{1}{5}$.

Some of this information is shown in the tree diagram below.

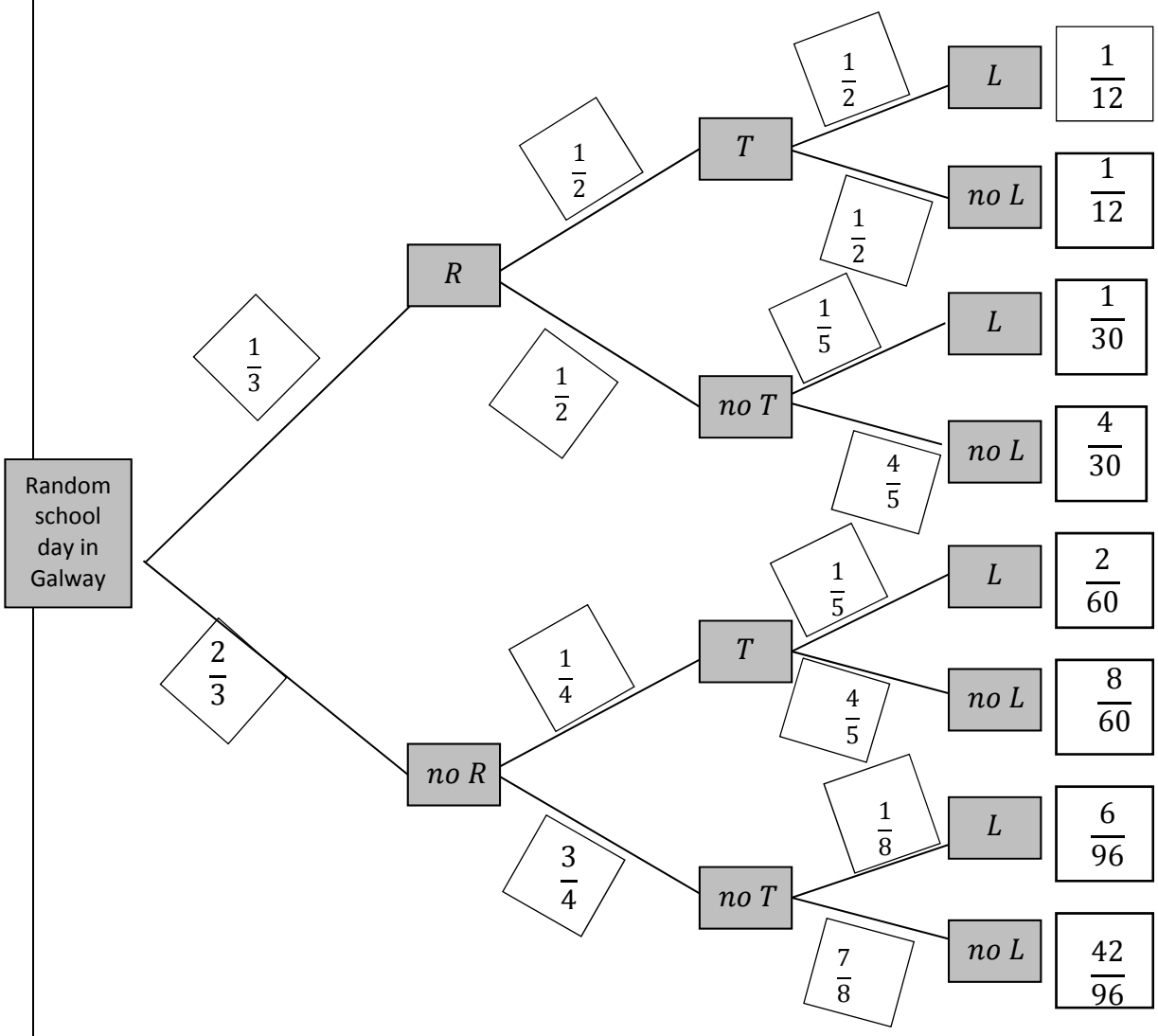
(i) Write the probability associated with each branch of the tree diagram **and** the probability of each outcome into the blank boxes provided.
Give each answer in the form $\frac{a}{b}$, where $a, b \in \mathbb{N}$.



(ii) On a random school day in Galway, find the probability of being late for school.

(iii) On a random school day in Galway, find the probability that it rained in the morning, given that you were late for school.

Q8	Model Solution – 60 Marks	Marking Notes
(a) (i)	$\mu = 63{\cdot}5 \qquad \sigma = 10$ $z = \dfrac{50 - 63{\cdot}5}{10} = -1{\cdot}35$ $P(z > -1{\cdot}35) = P(z < 1{\cdot}35)$ $= 0{\cdot}9115$ $91{\cdot}15\%$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • μ or σ identified *Mid Partial Credit:* • z found *High Partial Credit:* • $P(z < 1{\cdot}35)$and stops
(a) (ii)	$P(x > Z) = 0{\cdot}015$ $P(x < Z) = 0{\cdot}985$ $Z = 2{\cdot}17$ $\dfrac{x - 63{\cdot}5}{10} = 2{\cdot}17$ $x = 85{\cdot}2$ kg	**Scale 5D(0, 2, 3, 4, 5)** *Low Partial Credit:* • identifies $0{\cdot}985$ *Mid Partial Credit:* • identifies $2{\cdot}17$ *High Partial Credit:* • formula for x fully substituted
(a) (iii)	$n = 150, \qquad \bar{x} = 62, \qquad s = 10$ kg $H_o \rightarrow$ mean weight has not changed $H_1 \rightarrow$ mean weight has changed $z = \dfrac{62 - 63{\cdot}5}{\dfrac{10}{\sqrt{150}}}$ $= -1{\cdot}8371 > -1{\cdot}96$ Mean weight has not changed **or** Confidence interval: $\bar{x} \pm 1{\cdot}96\frac{\sigma}{\sqrt{n}}$ $62 \pm 1{\cdot}96\frac{10}{\sqrt{150}}$ $62 \pm 1{\cdot}96(0{\cdot}8165)$ $62 \pm 1{\cdot}6003$ $[60{\cdot}3997, 63{\cdot}6003]$ $63{\cdot}5$ falls within this interval $\therefore$ insufficient evidence to reject the null hypothesis The mean weight has not changed	**Scale 15D (0, 5, 7, 9, 15)** *Low Partial Credit:* • z formulated with some substitution • states null/alternative hypothesis only • reference to $\pm 1{\cdot}96$ *Mid Partial Credit:* • z fully substituted *High Partial Credit:* • $z = -1{\cdot}8371 > -1{\cdot}96$ • fails to contextualise the answer

(b) **(i)**	
(b) **(i)**		**Scale 15C (0, 6, 9, 15)** *Low Partial Credit:* • 3 boxes filled correctly *High Partial Credit:* • 10 boxes filled correctly

[37]

(b) (ii)	$\frac{1}{12}+\frac{1}{30}+\frac{2}{60}+\frac{6}{96}=\frac{17}{80}$ **or** $0{\cdot}2125$	**Scale 10C (0, 4, 5, 10)** *Low Partial Credit:* • 2 relevant fractions transferred *High Partial Credit:* • 4 relevant fractions identified but fails to complete
(b) (iii)	$P(R\|L)=\frac{P(R\cap L)}{P(L)}=\frac{\frac{1}{12}+\frac{1}{30}}{\frac{17}{80}}$ $=\frac{28}{51}$ **or** $0{\cdot}5490$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* • $P(L)$ • $P(R\cap L)$ *High Partial Credit:* • Formula fully substituted